A COMBINATORIAL APPROACH FOR OPTIMIZING TRANSPORTATION SYSTEM: MULTI-OBJECTIVE DECISION-MAKING FRAMEWORK

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Abstract: This study presents a comprehensive multi-objective transportation model aimed at optimizing complex vehicle routing problems, which are nondeterministic polynomial time NP-hard due to spatial, temporal, and capacity constraints. In this study, the multi-objective transportation model integrates decision-maker preferences with hybrid optimization techniques, including the approximate-combinatorial method, ant colony optimization and evolutionary algorithms. it seeks to minimize transportation costs, time, and emissions while accounting for real-world constraints such as fleet composition, customer demand, and service-level agreements. The techniques like multi-criteria decision-making methods are employed to refine the solution set, balancing objectives like cost, time, environmental impact, and service level. The novel optimization model is applied to a fuel distribution case study involving 18 customers and a heterogeneous fleet, where it optimizes vehicle routes to meet delivery requirements efficiently. The multi-objective transportation framework generates multiple feasible solutions, which are further narrowed down using decision-making frameworks to ensure alignment with organizational goals and decision-maker preferences. The integration of quantitative optimization techniques with qualitative decision-making processes makes this model robust and scalable, offering a practical tool for enhancing operational efficiency in transportation systems. This approach effectively addresses real-world logistics challenges, demonstrating significant improvements in route efficiency, cost savings, and environmental sustainability.

Key words: *decision making, emissions reduction, multi-objective transportation, operational efficiency, optimization, transportation costs, vehicle routing problem*

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1. Introduction

In its modern understanding, systems engineering, as well as engineering systems, differs from traditional disciplines by emphasizing several key principles. It treats systems as integrated wholes [3, 21, 7], aiming to harmonize their components, including functions, timelines, and objectives that may vary across different levels. Conceptual systemic design forms a core aspect of this discipline, alongside efforts to bridge traditional engineering fields and address gaps between specialized areas [3, 21, 7].

Modern systems engineering has evolved to redefine its domain and research methodologies to address the challenges posed by technological advancements, competitive pressures, and the complexity brought about by increasing automation and specialization. These challenges necessitate innovative strategies to effectively manage areas and interfaces across various disciplines.

Complex engineering systems are often exemplified by airport traffic control systems, the design and operation of sophisticated engineering installations, solutions to intricate vehicle routing problems tailored to real-world logistics scenarios [33, 15, 30], and passenger aircraft systems [20]. The systemic approach in these contexts seeks to achieve equilibrium, ensuring that no performance indicator improves at the expense of others of equal or greater importance [3, 27, 11]. For example, improving productivity should not result in unacceptable increases in cost or energy consumption. As performance indicators are inherently interdependent, decision-making must balance all critical parameters to optimize both the entire system and its individual components.

The optimization of decision-making processes in complex engineering systems requires a profound understanding of the efficiency indicators that govern the performance of decision-makers. In systems where multiple stakeholders influence decisions, their preferences and objectives often create intricate interdependencies that demand iterative optimization strategies. Traditional optimization techniques often prove inadequate for such multidimensional problems, necessitating the decomposition of complex tasks into smaller, more manageable sub-problems, each addressed with specialized approaches.

To address these challenges, the approximate-combinatorial method [16, 19, 3] provides a robust theoretical framework for decomposing and optimizing complex tasks. By breaking down a problem into a series of interconnected sub-tasks, this method facilitates iterative refinements to decision-makers' preference structures, progressively incorporating and expanding objectives. In this study, the approximate-combinatorial method is applied to the domain of transportation systems, with a particular focus on the canonical vehicle routing problem (VRP).

The VRP stands as a quintessential example of a combinatorial optimization problem, central to the field of transportation logistics and operations research [14, 24]. Introduced by Dantzig and Ramser [8], the VRP seeks to determine the most efficient set of routes for a fleet of vehicles tasked with delivering goods to a network of customers. The primary objective is to minimize total operational costs—commonly modeled as a combination of factors such as total distance traveled, fuel consumption, and time—while adhering to various constraints including vehicle capacities, time windows for deliveries, and the demand characteristics of individual customers [37].

The VRP's complexity stems from its combinatorial explosion of possible solutions as the number of vehicles and delivery locations increases. Mathematically, the VRP belongs to the class of NP-hard problems, where the time required to compute an optimal solution grows non-polynomially with the size of the problem [25]. In such problems, the solution space is vast, and exact algorithms—such as those based on branch and bound or dynamic programming—become impractical for large instances due to their exponential time complexity. Consequently, solving large-scale VRPs requires the application of heuristic or metaheuristic techniques, which offer approximations of the optimal solution within a reasonable computational time. Methods such as genetic algorithms [35], simulated annealing [2], or ant colony optimization [5] have been extensively studied and deployed in practice, given their ability to explore and exploit the search space efficiently.

In practical applications, the VRP often evolves into a multi-objective problem, driven by the need to balance several competing criteria. For instance, besides minimizing operational costs, decision-makers may aim to minimize the environmental impact of transportation, balance the workload among drivers, or ensure customer satisfaction by minimizing delivery times. This has led to the formalization of multi-objective vehicle routing problems (MoVRPs), wherein multiple objectives must be optimized simultaneously, often with trade-offs that cannot be fully reconciled by a single solution. As such, Pareto-optimality becomes a key concept in multi-objective optimization, where a solution is deemed optimal if no objective can be improved without deteriorating at least one other objective. MoVRPs are an active area of research, with contributions from a variety of authors [29, 36, 18, 17, 47, 45, 22, 13, 40, 42, 10, 26], each proposing methods to model and solve these problems through a range of techniques, from evolutionary algorithms to hybrid metaheuristics.

However, the challenge of expressing and formalizing the intricate interrelationships between decision-makers' preferences, customer demands, and the structural characteristics of the distribution process remains unresolved. These interrelationships form a highly nonlinear and multi-objective optimization landscape that is sensitive to both real-time data inputs and subjective human judgment [23]. This study aims to address this challenge by integrating decision-makers' preferences into a mathematically rigorous optimization framework that reflects the complexity of real-world transportation systems. Through the adoption of a multi-objective optimization approach, we align our modeling framework with the latest international research trends in combinatorial optimization, developing solutions that are computationally feasible and practically meaningful.

Transportation systems also exhibit a dual complexity arising from the interplay between continuous and discrete decision-making problems. Continuous problems, such as those involving cost minimization or time optimization, often coexist with discrete challenges, such as determining vehicle allocation or route selection. To manage this, decision-makers rely on multiple models that offer different levels of abstraction and granularity. These include multiple objective decision optimization (MODO) models [4, 32], which handle continuous and quantifiable objectives; multiple criteria decision-making (MCDM) models, which are tailored for discrete, low-dimensional problems; and Subjective Models (SM), which capture non-formalized, experience-based knowledge and human intuition. The integration of these models enables a more holistic approach to transportation optimization, combining the quantitative rigor of MODO and MCDM models with the flexibility of SM, which accounts for unforeseen or unmodeled factors in real-world decision-making.

To address critical challenges in the optimization of transportation systems, this research focuses on advancing the VRP through a multi-objective approach. The primary motivation lies in balancing conflicting objectives such as cost minimization, environmental sustainability, and adherence to operational time constraints. Existing methodologies often emphasize single-objective optimization or fail to accommodate the complexities of real-world conditions, including fleet capacity limitations and diverse customer demands. To bridge these gaps, we propose a multi-objective transportation (MOT) model that integrates decision-maker preferences, incorporates practical constraints, and leverages advanced optimization techniques. By doing so, the model aims to offer a holistic and actionable framework for addressing transportation challenges effectively.

The structure of this paper reflects a systematic approach to achieving these goals. The theoretical foundations of multi-objective optimization methods and the hybrid techniques utilized, such as ant colony optimization (ACO) and evolutionary algorithms, are explored in detail. The practical application of the MOT model is demonstrated through a case study involving a fuel distribution problem that considers a heterogeneous fleet and 18 customers. The analysis explores the implications of the proposed approach, offering insights into its broader effectiveness and applicability.

2. Methods

2.1 Approximate Combinatorial Method

The approximate combinatorial method provides a framework for addressing complex optimization tasks by decomposing them into a series of simpler subproblems. This approach is particularly useful in multi-objective optimization tasks, where direct solutions are often infeasible due to the high-dimensional decision space and the complexity of the involved objective functions. In this paper, the approximate combinatorial method is applied to optimize decision-makers' preferences in transportation systems, specifically focusing on the VRP. By iteratively refining the search space and approximating the objective functions, the method offers a practical way to handle large-scale optimization problems.

Given the set of possible solutions D, the objective function vector is defineda as

$$Z(x) = (z_1(x), \dots, z_n(x)), \tag{1}$$

for a complex task that needs to be solved. The goal is to find a solution $x^* \in D$ such that:

$$Z(x^*) = \min Z(x), \quad x \in D. \tag{2}$$

Practically, any multi-objective optimization task can be formulated in this way. The following general procedure is used.

In the set D, an approximation function vector $Q(x)$ is defined such that:

$$Z(x^*) \geq Q(x^*), \tag{3}$$

and for $Q(x)$, effective methods and algorithms exist for determining not only $x_0 \in D$:

$$Q(x_0) = \min Q(x), \quad x \in D, \tag{4}$$

but also for all elements $x \in D$ with $Q(x)$ values that differ from $Q(x_0)$ by no more than a parameter vector $\alpha \geq 0$. In this way, it is assumed that it becomes possible to determine a set of solutions $D_0 \subset D$ such that:

$$Q(x_0) \leq Q(x) \leq Q(x_0) + \alpha \implies x \in D_0, \tag{5}$$

$$Q(x) > Q(x_0) + \alpha \implies x \in D - D_0. \tag{6}$$

Among other consequences derived from the decomposition principles of the method, the solution search is performed through successive approximations. This is achieved by solving a series of approximate optimization tasks with objective functions $Q_1(x), Q_2(x), \ldots, Q_m(x)$ in D, such that:

$$Z(x^*) \geq Q_1(x^*) \geq Q_2(x^*) \geq \cdots \geq Q_m(x^*), \tag{7}$$

where $Q_i(x)$ represents an approximation of $Q_{i-1}(x)$. Thus, it becomes possible to search for the optimal solution to the original task within a series of progressively smaller solution subsets (populations), beginning from D_0^n:

$$D_0^0 \subset D_0^1 \subset \cdots \subset D_0^n \quad \text{and} \quad Z(\bar{x}) = Z(x^*), \tag{8}$$

where $\bar{x}$ is the optimal solution of $Z(x)$ in D_0^0.

In more general cases, if the solution set generated while searching for solutions to those subtasks is incomplete, and the selection of the best solution composition is considered "optimal," the procedure becomes an approximate optimization process starting from solution populations obtained through heuristic methods. Consequently, the entire procedure is heuristic. It is important to note that generating solution populations by heuristic evolutionary methods presupposes the generation of α-optimal options. Although these solutions are generally incomplete, they facilitate finding approximate solutions to optimization tasks by successive approximations.

2.2 Utility Functions Approximation

When addressing complex systems like the VRP, which involve multiple, often conflicting objectives, it is typically computationally infeasible to optimize all criteria simultaneously. To manage this complexity, the approximate combinatorial method is employed, allowing the problem to be decomposed into smaller, more tractable subproblems. In this context, a partial utility function that considers only a subset of the criteria can serve as an approximation of the full utility function. This iterative approach enables optimization by progressively adding criteria and refining the solution.

In this section, we focus on the approximation of the utility function $U(z_1, \ldots, z_r)$, which initially considers only r criteria. This utility function is treated as an approximation of the complete utility function $U(z_1, \ldots, z_n)$, which includes all n criteria. The objective is to develop an efficient approximation that facilitates convergence towards an optimal solution for all n criteria, by iteratively expanding the criteria space and adjusting the utility function.

Let the decision-maker's preference system be represented by the utility function $U(z_1, \ldots, z_r)$, which incorporates the first r criteria of the multi-objective optimization task. This function constitutes an approximation of the full utility function $U(z_1, \ldots, z_n)$, which includes all n criteria. Under these conditions, the following inequality holds:

$$U(z_1^0, \ldots, z_r^0) \le U(z_1^*, \ldots, z_n^*), \tag{9}$$

where $(z_1^0, \ldots, z_r^0)$ is the solution vector minimizing $U(z_1, \ldots, z_r)$ over $x \in D$, and $(z_1^*, \ldots, z_n^*)$ is the solution vector minimizing $U(z_1, \ldots, z_n)$ over $x \in D$. The reason for this relationship is that $U(z_1^*, \ldots, z_r^*)$ constitutes the projection of the efficient, non-dominated solution $U(z_1^*, \ldots, z_n^*)$ from the n-dimensional criteria space (variables $z_1, \ldots, z_n$) onto the smaller r-dimensional criteria space (variables $z_1, \ldots, z_r$). Thus, the approximate-combinatorial method ensures that the function $U(z_1(x), \ldots, z_r(x))$ is an approximation of $U(z_1(x), \ldots, z_n(x))$, regardless of whether the criteria $z_{r+1}, \ldots, z_n$ are quantitative or subjective.

It should be noted that the solutions $(x_1^0, \ldots, x_r^0)$ and $(x_1^*, \ldots, x_r^*)$ are generally different. Adding new criteria necessitates renewing the "optimal" solution $(x_1^0, \ldots, x_r^0)$ with another solution $(x_1^*, \ldots, x_r^*)$. Depending on the methods used for evaluating the criteria $z_{r+1}, \ldots, z_n$, there exists an α value such that the optimal value of the task can be found within the α-optimal solutions for the approximate utility function task:

$$\min_{x \in D} U(z_1(x), \ldots, z_r(x)), \tag{10}$$

while simultaneously satisfying the given constraints. Due to the abstract nature of the utility function, practical problems require substituting this function with approximations, possibly defined through numerical procedures.

2.3 Approximation and Multi-Criteria Decision-Making

Relation (9) implies that $U(z_1, \ldots, z_r)$ is an approximation of $U(z_1, \ldots, z_n)$. Consequently, the final population obtained from optimizing a task with only a subset of the original criteria can be processed using more complex procedures that consider all the original criteria and reordering them accordingly. This makes it possible to find a population with the aid of an optimization model and apply MCDM methods to the final population considering all the criteria involved.

Thus, the solution to the decision-making process can be reduced to solving the task:

$$x^* = \arg\min_x U(z_1(x), \ldots, z_n(x)), \quad x \in D. \tag{11}$$

This can be approximated by:

$$U(z_1(x_0), \ldots, z_r(x_0)) = \min_x \{U(z_1(x), \ldots, z_r(x))\}, \quad x \in D, \quad n \geq r. \tag{12}$$

According to the approximate-combinatorial method and the property of any utility function, the solution of the task:

$$U(z_1(x^*), \ldots, z_n(x^*)) = \min_x \{U(z_1(x), \ldots, z_n(x))\}, \quad x \in D, \tag{13}$$

can be found within the solution population given by:

$$U(z_1(\bar{x}), \ldots, z_r(\bar{x})) = \min\{U(z_1(x) + \alpha_1, \ldots, z_r(x) + \alpha_r)\}, \quad x \in D. \tag{14}$$

If all components of $\alpha = (\alpha_1, \ldots, \alpha_r)$ are sufficiently large, then:

$$z_1(\bar{x}) + \alpha_1, \ldots, z_r(\bar{x}) + \alpha_r \geq z_1(x^*), \ldots, z_r(x^*), \tag{15}$$

where x^* is the optimal solution of the task in the solution subspace $D_0 \subset D$.

The vector α represents the loss of efficiency required for the first r objectives in order to obtain the best trade-off between all n objectives in the decision-making problem. In other words, α captures the acceptable deviation from the optimal values for the first r objectives to achieve a balanced solution considering all criteria.

In cases where the error in determining the solution is given by:

$$z_1(\bar{x}) + \alpha_1, \ldots, z_n(\bar{x}) + \alpha_r \leq z_1(x), \ldots, z_n(x) \leq z_1(x^*), \ldots, z_r(x^*), \tag{16}$$

the utility function can be approximated by a series of tasks, including multi-objective optimization, MCDM procedures, or subjective solution selection from a population.

In cases where exact methods are used to solve multi-objective optimization tasks, relations (13)–(16) have strict character. However, when metaheuristic methods are applied, they provide an evaluation of the required parameters $\alpha = C - Q(x_0)$, where $C = z_i^{\sup}$. The population size must ensure that relation (14) is satisfied, and the α components are not known in advance. For a fixed-size evolving population, the ith component of α can be evaluated as:

$$z_i < z_i^{\sup}, \tag{17}$$

and

$$\alpha_i = z_i^{\sup} - z_i^{inf}, \quad \forall i = 1, \ldots, r, \tag{18}$$

where $z_i^{\sup}$ is the highest value of the z_i objective in the final population and z_i^{inf} is the lowest value.

Moreover, as a result of searching for the best trade-off in decision-making, the objectives $z_{i+1}, \ldots, z_n$ will also receive values between their possible lower and upper bounds. Thus, relations (17) and (18) can be extended to objectives $z_{i+1}, \ldots, z_n$.

If α is calculated for a given population using relations (17) and (18), the closeness of the utility function's optimality is determined by the precision of the metaheuristic procedure and the quality of the MCDM modeling, rather than by the objectives reconciliation procedure.

2.4 Multi-Objective Transportation Problem Definition

MOT refers to the process of satisfying transportation demands while considering decision-makers' preferences and utilizing available company resources. It involves determining the most feasible set of routes from a central depot to a geographically dispersed set of clients, subject to various constraints, thus facilitating logistical decision-making in the distribution process. The results provide crucial insights into which vehicle should undertake a given route, how to meet client demands, allocate cargo, and sequence the routes.

The problem addressed in this research expands on the standard definition of MOT, incorporating a series of conditions that refine transportation models for real-world scenarios. These models adapt to the process characteristics such as communication routes, customer demands, fleet characteristics, and the transportation process itself. The precision in modeling these characteristics allows for generalization across transportation processes with the following considerations:

- Asymmetric and deterministic VRP: The underlying graph is a directed graph, indicating that the distance between two points is not necessarily the same in both directions.
- Predefined customer time windows: Each customer must be serviced within a specific time window.
- Heterogeneous, fixed, and compartmentalized fleet: The fleet consists of vehicles with different capacities, costs, and compartmentalized structures.
- Compatibility constraints: These exist between fleet and customers, fleet and products, and between different products themselves.
- Multiple trips per vehicle: Vehicles are allowed to make several trips to meet demand.
- Decision-makers' priorities: Decision-makers intervene to set priorities for customers and products.
- Multiple preferences/objectives: The optimization seeks to balance multiple objectives, which may conflict with each other.

The transportation problem is modeled as a directed graph $G(V, A)$. Let $V = \{0\} \cup N$ represent the set of vertices, where $N = \{1, 2, \ldots, n\}$ denotes the customers and 0 represents the depot. In this case, the distances between some pairs of vertices may be asymmetric, meaning that $d_{ij} \neq d_{ji}$ for at least one pair $i, j \in V$. Therefore, the problem is formulated on a directed graph $G = (V, A)$ with a set of arcs $A = \{(i, j) \in V \times V : i \neq j\}$, and the arc distances d_{ij} for $(i, j) \in A$.

The transportation requests involve the distribution of products from the depot (point 0) to a set of n customers. Each customer $i \in N$ has a known demand $q_i = (q_{i1}, q_{i2}, \ldots, q_{ip}) \geq 0$, where p represents the number of products. The depot itself has no demand, indicated by $q_0 = 0$.

Customer demands must be satisfied within predefined time windows $[a_i, b_i]$. The service time at the depot is denoted by ς^0_{pkt}, representing the loading operation.

Serving customer i takes time ς^i_{pkt} and must begin within the specified time window. Arriving earlier than a_i results in a waiting time ϵ^i_{kt}, while service must always be completed before b_0. Similarly, all operations must start after a_0, and vehicles must return to the depot before b_0.

The fleet is heterogeneous, consisting of $|K|$ vehicles, each with different characteristics and associated costs. Each vehicle $k \in K$ is divided into compartments $C = \{1, 2, \ldots, c\}$, with compartment capacities Q_{ck}. For each arc $(i, j) \in A$ and vehicle $k \in K$, the following parameters are defined: travel cost c^t_{ijk}, travel time τ^t_{ijk}, and emissions e^t_{ijk}. Additionally, the profit generated by each vehicle depends on the revenue from the sale price μ_p of the load q_{ip}, and the expenses associated with this load, denoted as γ_p.

Each vehicle $k \in K$ serves a subset of customers $V_k \subseteq V$, based on fleet-customer compatibility constraints. Each compartment c of vehicle k may be dedicated to a specific type of product, especially in cases where product incompatibilities exist. Furthermore, each vehicle can transport a set of products $P_k \subseteq P$, adhering to fleet-product compatibility constraints.

Due to fleet limitations, vehicles may need to make multiple trips across different transportation periods $T = \{1, 2, \ldots, t\}$. A vehicle serving a subset of customers $S \subseteq N$ starts from the depot, visits each customer in S, and returns to the depot. If required, the vehicle can refuel at the depot and then resume its next route. A route (or trip) is defined as (r, k, t), where $r = (i_0, i_1, \ldots, i_s, i_{s+1})$ is a sequence with $i_0 = i_{s+1} = 0$, and the set $S = \{i_1, \ldots, i_s\} \subseteq N$ contains the customers to be visited. A route (r, k, t) is feasible if both capacity and time constraints are satisfied, making S a feasible cluster.

A solution to the MOT problem consists of $|K|$ feasible routes, with at least one route per vehicle. The routes $r_1, r_2, \ldots, r_{|K|}$ and corresponding clusters $S_1, S_2, \ldots, S_{|K|}$ form a feasible solution to the VRP if all routes are feasible and the clusters partition the set N. The VRP thus involves two interconnected tasks:

- Partitioning the set of customers N into feasible clusters $S_1, \ldots, S_{|K|}$;
- Routing each vehicle $k \in K$ through $\{0\} \cup S_k$.

Decision-makers intervene by setting priorities at two levels: prioritizing customers to visit, denoted as ϕ_i, and prioritizing products to distribute, where ϕ_i may also define customers who demand prioritized products.

The MoVRP, like the classic VRP, is an NP-hard problem. As the number of variables—such as vehicles, customers, and constraints—increases, the complexity grows exponentially, making it computationally infeasible to find optimal solutions for large instances. Exact optimization methods, such as branch-and-bound or dynamic programming, become impractical due to the combinatorial explosion of potential solutions. Consequently, heuristic and metaheuristic approaches are necessary to provide near-optimal solutions within a reasonable computational timeframe.

The focus therefore shifts to optimizing multiple conflicting objectives based on a preference system. Key objectives include total transportation cost $z_1(x)$, total profit-loss $z_2(x)$, total transportation time $z_3(x)$ and environmental pollution from transportation $z_4(x)$. In addition, several multi-criteria selection objectives are considered, i.e. capacity utilization $z_5(x)$, total sales $z_6(x)$, total distance $z_7(x)$ and service level $z_8(x)$. Lastly, decision-makers consider the operator's experience with real-world situations, represented by $z_9(x)$.

The notation used in the optimization model is presented in Tab. I. The first four efficiency indicators are selected as objective functions to describe decision-makers' preferences related to cargo transportation. These indicators are grouped into four dimensions: economic, temporal, environmental, and spatial (the latter being implicit in the modeling).

$$z_1(x) = \sum_{k \in K} f_k \sum_{t \in T} \sum_{j \in V \setminus \{0\}} x_{0jk}^t + \sum_{t \in T} \sum_{k \in K} \sum_{(i,j) \in A} c_{ijk}^t x_{ijk}^t, \tag{19}$$

$$z_2(x) = -\sum_{p \in P} \sum_{t \in T} \sum_{k \in K} \sum_{i \in V \setminus \{0\}} (\mu_p - \gamma_p) q_{ip} y_{ipk}^t, \tag{20}$$

$$z_3(x) = \sum_{t \in T} \sum_{k \in K} \left[\sum_{(i,j) \in A} \tau_{ijk}^t x_{ijk}^t + \sum_{p \in P} \sum_{i \in V} (\varsigma_{ipk}^t + \epsilon_{ik}^t) y_{ipk}^t \right], \tag{21}$$

$$z_4(x) = \sum_{t \in T} \sum_{k \in K} \sum_{(i,j) \in A} e_{ijk}^t x_{ijk}^t, \tag{22}$$

where $x_{ijk}^t = 1$ if the arc $(i,j) \in A$ is traversed by vehicle k in period t, and $y_{ipk}^t = 1$ if client i is visited by vehicle k with product p in period t. The term $\sum_{t \in T} \sum_{j \in V \setminus \{0\}} x_{0jk}^t$ represents the number of vehicles used in period t.

These objective functions can be broken down as follows:

- The total transportation cost objective function (19) is decomposed into fixed and variable costs.
- The profit/loss function (20) is derived from total income and expenses.
- The total transportation time (21) includes travel, service, and waiting times.
- Environmental pollution (22) is directly linked to atmospheric emissions of greenhouse gases.

This problem entails a series of conditions subject to a system of constraints. The constraints can be classified into structural, spatial, and temporal categories, which characterize vehicle routing problems with heterogeneous fleets, compartments, and multiple trips. The relevant constraints are defined as follows:

Input data characterizing the process	
N	Set of nodes or clients $= \{1, 2, \ldots, n\}$
V	Set of vertices $= \{0\} \cup N$, where 0 is the depot
A	Set of arcs $= \{(i, j) \in V \times V : i \neq j\}$
C	Set of compartments $= \{1, 2, \ldots, c\}$
T	Set of transportation periods $= \{1, 2, \ldots, t\}$
S	Set of visited clients $= \{i_1, \ldots, i_s\} \subseteq N$
d_{ij}	Distance from i to j (km)
μ_p	Selling price of product p (monetary)
γ_p	Cumulative cost of product p (monetary)
u_{ik}	1 if client $i \in V_k$, 0 otherwise (binary)
v_{pk}	1 if product $p \in P_k$, 0 otherwise (binary)
f^t_{pck}	1 if compartment c can transport product p in vehicle k in period t, 0 otherwise (binary)
Coordination variables	
q_{ip}	Product p to be delivered to client i (volume/weight/units)
K	Set of vehicle fleet $= \{1, 2, \ldots, k\}$
P	Set of products to distribute $= \{1, 2, \ldots, p\}$
Q_{ck}	Capacity of compartment c of vehicle k (volume/weight/units)
a_i	Lower bound of the time window to service client i (hour)
b_i	Upper bound of the time window to service client i (hour)
Efficiency indicators	
$z_1(x)$	Total transportation cost (TC)
$z_2(x)$	Total profit-loss of transportation (TPr)
$z_3(x)$	Total transportation time (TT)
$z_4(x)$	Total transportation pollution (TP)
$z_5(x)$	Capacity utilization (CU)
$z_6(x)$	Total sales (TS)
$z_7(x)$	Total distance (TD)
$z_8(x)$	Service level (SL)
$z_9(x)$	Operator experience with real situations
Intermediate variables of interest	
c^t_{ijk}	Travel cost of vehicle k moving from i to j in period t (monetary)
τ^t_{ijk}	Travel time of vehicle k from i to j in period t (minutes)
ς^t_{ipk}	Service time of vehicle k with product p for client i in period t (minutes)
ϵ^t_{ik}	Waiting time of vehicle k to service client i in period t (minutes)
e^t_{ijk}	Emissions of vehicle k moving from i to j in period t (CO2-equivalent)
σ_i	Start of service time at client i
ϕ_i	Priority assigned by the decision-maker to client i
Decision variables	
w^t_{pck}	1 if product p is assigned to compartment c of vehicle k in period t, 0 otherwise (binary)
x^t_{ijk}	1 if the arc $(i, j) \in A$ is traveled by vehicle k in period t, 0 otherwise (binary)
y^t_{ipk}	1 if client i is visited by vehicle k with product p in period t, 0 otherwise (binary)

Tab. I *Extended notation for multi-objective transportation.*

$$\sum_{j \in V \setminus \{0\}} x_{0jk}^t \leq 1 \quad k \in K, t \in T, \tag{23}$$

$$\sum_{j \in V, i<j} x_{ijk}^t + \sum_{j \in V, j<i} x_{jik}^t = 2y_{ipk}^t \quad i \in V \setminus \{0\}, p \in P, k \in K, t \in T, \tag{24}$$

$$\sum_{(i,j) \in A(S)} x_{ijk}^t \leq |S| - 1 \quad S \subseteq V \setminus \{0\}, |S| \geq 2, k \in K, t \in T, \tag{25}$$

$$\sum_{p \in P} w_{pck}^t \leq 1 \quad c \in C, k \in K, t \in T, \tag{26}$$

$$\sum_{i \in V \setminus \{0\}} \sum_{p \in P} q_{ip} y_{ipk}^t \leq Q_{ck} w_{pck}^t \quad p \in P, k \in K, t \in T, \tag{27}$$

$$\sum_{i \in V \setminus \{0\}} y_{ipk}^t \leq |V| \cdot \sum_{c \in C} w_{pck}^t \quad p \in P, k \in K, t \in T, \tag{28}$$

$$w_{pck}^t \leq f_{pck}^t \quad p \in P, c \in C, k \in K, t \in T, \tag{29}$$

$$w_{pck}^t \in \{0, 1\} \quad p \in P, c \in C, k \in K, t \in T, \tag{30}$$

$$x_{ijk}^t \in \{0, 1\} \quad (i, j) \in A, k \in K, t \in T, \tag{31}$$

$$y_{ipk}^t \in \{0, 1\} \quad i \in V \setminus \{0\}, p \in P, k \in K, t \in T, \tag{32}$$

$$f_{pck}^t \in \{0, 1\} \quad p \in P, c \in C, k \in K, t \in T. \tag{33}$$

Constraint (23) ensures that a vehicle departs at most once per trip. Constraints (24) and (25) implement flow preservation and subtour elimination, respectively. Each vehicle compartment can only be dedicated to one product type (26). The total quantity of product a vehicle can service in a single trip is limited by the compartment capacities (27). Constraint (28) connects the variables y_{ipk}^t and w_{pck}^t. Constraint (29) ensures that products are assigned to appropriate compartments, while constraints (30)–(33) enforce non-negativity and integrality for the decision variables.

From a temporal perspective, the transportation system is subject to the following constraints:

$$\sigma_i + (\tau_{ijk}^t + \varsigma_{ipk}^t + \epsilon_{ik}^t) x_{ijk}^t - M(1 - x_{ijk}^t) \leq \sigma_j \quad (i, j) \in A, p \in P, k \in K, t \in T, \tag{34}$$

$$a_i \leq (\sigma_i + \varsigma_{ipk}^t + \epsilon_{ik}^t) \leq b_i \quad i \in V, p \in P, k \in K, t \in T. \tag{35}$$

Constraint (34) ensures that the start of service at node j depends on the start of service time σ_i at node i, the service time ς_{ipk}^t, the waiting time ϵ_{ik}^t, and the travel time τ_{ijk}^t. Meanwhile, constraint (35) guarantees that service at each node occurs within the established time window. Here, M represents a sufficiently large positive number.

Considering the compatibilities defined in the model, the following constraints arise:

$$x_{ijk}^t \leq u_{ik} \quad (i, j) \in A, i \in V \setminus \{0\}, k \in K, t \in T, \tag{36}$$

$$w_{pck}^{t} \leq v_{pk} \quad p \in P, c \in C, k \in K, t \in T, \tag{37}$$

$$v_{pk} \in \{0,1\} \quad p \in P, k \in K, \tag{38}$$

$$u_{ik} \in \{0,1\} \quad i \in V \setminus \{0\}, k \in K. \tag{39}$$

Here, constraint (36) states that each vehicle can only visit nodes to which it is compatible, while constraint (37) limits the products that vehicle k can transport. Constraints (38) and (39) establish the non-negativity and integrality conditions for the respective variables.

In this model, two levels of priorities are established by decision-makers for transportation, i.e. priority over the clients to visit and priority over the products to distribute:

$$\sum_{j \in V, i \neq j} x_{ijk}^{t} \phi_j \geq \phi_j y_{ipk}^{t} \quad i \in V \setminus \{0\}, p \in P, k \in K, t \in T. \tag{40}$$

2.5 Solution Methodology for Optimal Operation of the Transportation System

As was mentioned, the research is based on the application of the approximate-combinatorial method [16, 19, 3], incorporating MODO (first solutions approximation), MCDM (second solutions approximation), and SM (third solutions approximation). These methods are employed to obtain a solution that best satisfies the decision-makers' preferences. The preference system is reflected in the utility function $U(z_1, z_2, \ldots, z_n)$, which considers all n criteria for evaluating the solutions. This utility function is approximated through multi-objective optimization, multi-attribute optimization procedures, and/or the subjective selection of solutions from the solution population.

The three-tiered solution approximation was meticulously designed to address the multidimensional complexity of the MOT problem. The first tier, MODO, employs ACO to explore the solution space comprehensively, generating initial feasible solutions that balance key dimensions such as cost, time, and environmental impact while maintaining computational efficiency for large-scale problems.

The second tier, MCDM, refines these solutions by incorporating additional criteria, including capacity utilization, service levels, and transportation distance. Methods such as PROMETHEE, AHP, and TOPSIS are used to align the solutions with decision-makers' specific preferences and priorities.

The third tier, Strategic Modeling (SM), integrates qualitative insights derived from decision-makers' real-world experience to address operational uncertainties, such as accidents or shifting customer priorities. This ensures that the final solutions are not only optimal but also practical and implementable.

This sequential approach transitions solutions from mathematically robust approximations to actionable decisions, offering a comprehensive framework for real-world optimization.

2.5.1 First Solutions Approximation with MODO

In the first approximation, the ACO algorithm is used (see Fig. 1). ACO is part of the class of swarm intelligence algorithms, modeled after the behavior of ants and other social insects. In this adaptation for the transportation system, multiple colonies are generated, each representing different types of vehicles at the depot (with varying characteristics and compatibilities). This design follows the contributions of Kubil et al. [22].

Artificial ants are placed at the depot and choose the next client to visit based on a probability rule:

$$p_i^k(t) = \frac{[\tau_{ix}(t)]^\alpha [\eta_{ix}]^\beta}{\sum_{x \in X} \sum_{i \in V_k} [\tau_{ix}(t)]^\alpha [\eta_{ix}]^\beta} \tag{41}$$

After visiting a node, the ant adds it to a taboo list, reduces the vehicle's compartment capacity Q_{ck} by the client's product demand q_{ip}, and assigns products to vehicle compartments. If $Q_{ck} > 0$, the ant continues selecting the next client

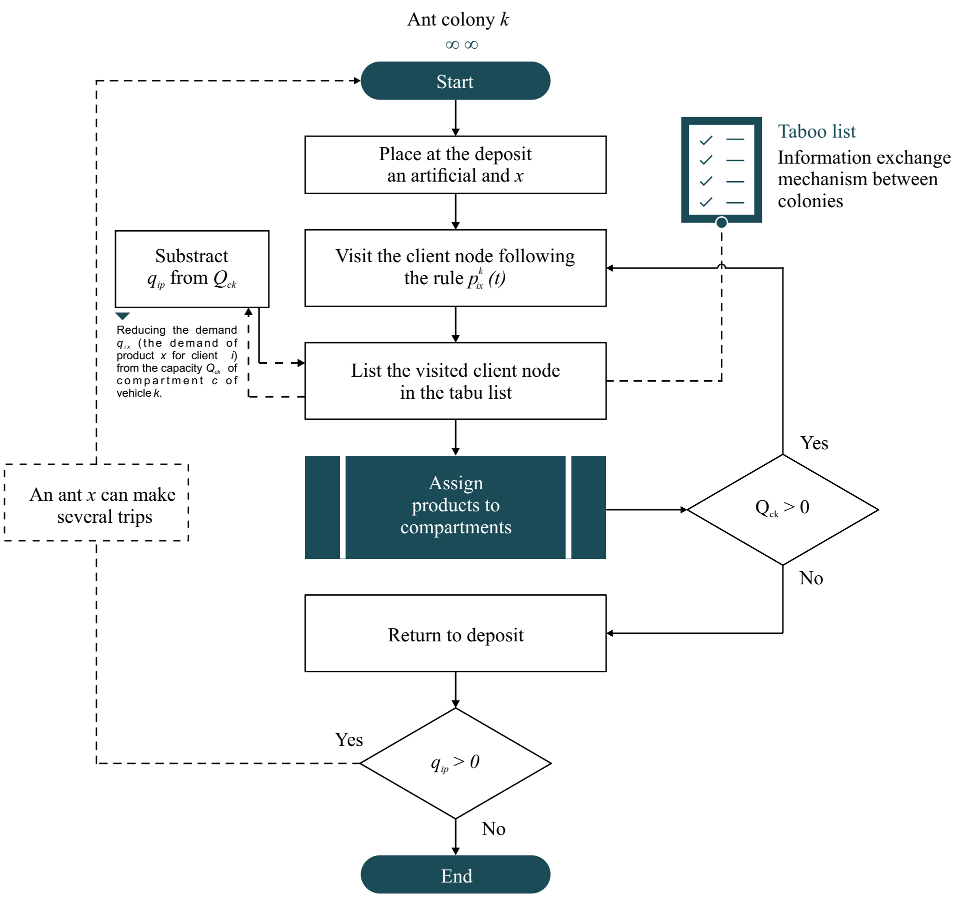


Fig. 1 *Algorithm for the initial population generation with ant colony optimization.*

using the probability rule; otherwise, it returns to the depot. This process is repeated until transportation demands are fulfilled.

This approach is unique in its ability to handle multiple compartments, assigning products to compartments during the route. The taboo list serves as a mechanism for exchanging information between colonies, constantly updating product demand and time availability at each node. This mechanism prevents vehicles from visiting already explored nodes, promoting exploration of new areas in the search space and avoiding redundant efforts. Attributes such as solution quality are incorporated to prioritize solutions with high fitness scores, ensuring efficient propagation of information and balanced exploration and exploitation within the algorithm.

The model (19)–(40) requires an ad hoc solution strategy combining evolutionary algorithms for multi-objective optimization. The goal is to iteratively evolve the set of feasible solutions toward a population of non-dominated, diverse, and uniformly distributed solutions. The evolutionary strategy incorporates the following stages:

1. Generation of an initial solutions population.
2. Selection of the set of feasible solutions.
3. Crossover between the best solutions.
4. Solution attribute mutation.
5. Preservation of the best solutions.

Steps 3 and 4 form the recombination mechanism. The initial population is generated using a modified ACO algorithm. The evolutionary algorithm then evolves this population by selecting solutions that dominate others, according to the dominance criterion evaluated in the fitness functions. Above mentioned evolutionary strategy is graphically depicted in Fig. 2.

2.5.2 Second Solutions Approximation with MCDM

In this second approximation, objectives such as $z_5(x)$ (capacity utilization), $z_6(x)$ (total savings), $z_7(x)$ (total time), $z_8(x)$ (total distance), and $z_9(x)$ (service level) are considered, alongside the first four indicators. While ACO focuses on optimizing economic, spatial, temporal, and environmental dimensions, MCDM methods complement MODO by providing a more detailed evaluation of the decision-makers' preference system, accounting for additional criteria that influence transportation performance.

One of the key methods used in this stage is the preference ranking organization method for enrichment evaluation (PROMETHEE) II method for global ranking. This approach involves several sequential steps. Initially, the criteria and the set of alternatives for the decision problem are defined. Following this, the weight of each criterion is determined, and the decision matrix is normalized to address both profit and cost criteria. After normalization, pairwise comparisons are conducted to identify deviations, and a preference function is established. Based on these

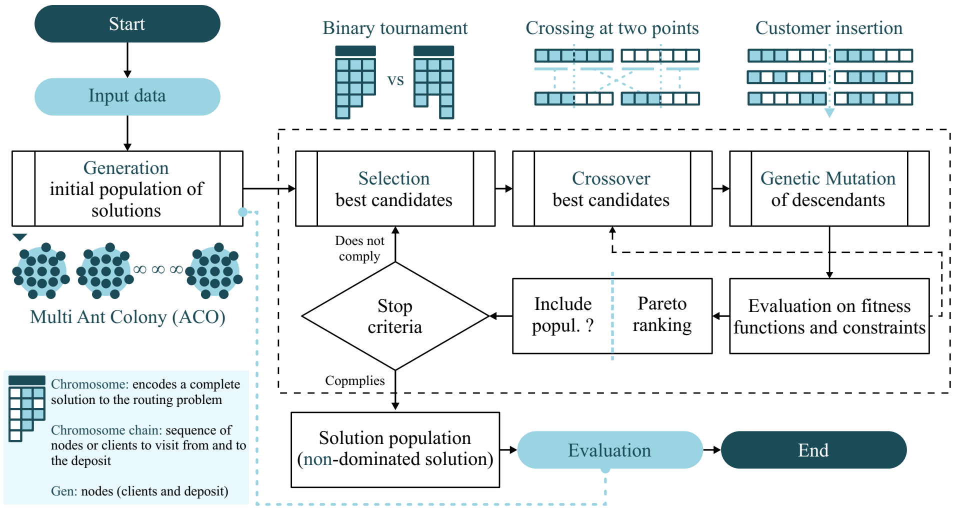


Fig. 2 *Evolutionary strategy for solving the multi-objective transportation model.*

assessments, the multicriteria index is computed, followed by the calculation of positive and negative flows. Ultimately, the net flow value is determined, which enables the ranking of alternatives.

In parallel, the the analytic hierarchy process (AHP) method [34] is applied to determine the criteria weighting. The process begins by defining a hierarchical model for the MOT problem. Pairwise comparisons are performed at each hierarchical level using the Saaty scale, followed by the construction of a normalized decision matrix. The priority vector for each criterion is then calculated, with the consistency index and consistency radius being computed to ensure the validity of the pairwise comparisons. Once consistency is verified, the individual priorities are aggregated into a group priority vector, reflecting the collective judgment of the decision-makers.

To complement these methods, the technique for order of preference by similarity to ideal solution (TOPSIS) method [38] is employed for solution selection. This process starts with the construction of a decision matrix, in which the attributes are transformed into a non-dimensional form, allowing for direct comparison. Next, a weighted normalized decision matrix is created based on the criteria weights assigned by the decision-makers. The positive ideal solution and the negative ideal solution are identified, and separation measurements from these ideal alternatives are calculated. The relative proximity of each alternative to the ideal solution is then determined, enabling the ranking of alternatives according to their closeness to the ideal solution.

Upon completing these analyses, the best alternative is presented to the decision-makers. Their satisfaction with the selected solution is assessed, and its applicability and effectiveness within the transportation process are evaluated. This comprehensive evaluation ensures that the chosen solution aligns with the objectives and preferences of the decision-makers while also addressing the constraints of the transportation system.

2.5.3 Third Solutions Approximation with Subjective Model

In this final step, the solutions from the second approximation are further refined through the application of SM, which take into account decision-makers' personal experience and expert judgment. Unlike the previous approximations, which rely primarily on mathematical and objective optimization techniques, the third approximation integrates qualitative factors that may not be easily modeled but are critical for practical decision-making in transportation systems.

Real-world conditions—such as unexpected accidents, adverse weather, and other unforeseen circumstances—often impact the operational efficiency and feasibility of transportation plans. These factors, which are difficult to predict or include in traditional optimization models, are crucial in determining the ultimate effectiveness of any proposed solution. Therefore, the decision-makers' personal insights and situational awareness are incorporated into this final approximation.

In this phase, the operator, leveraging both experience and real-time knowledge, plays a decisive role. The operator reviews the solutions generated in the second approximation, focusing on how well they align with the efficiency indicators (such as cost, time, capacity utilization, environmental impact, and service levels) and the specific context of the transportation task. The subjective nature of this stage allows the operator to account for nuanced operational realities that mathematical models may overlook.

The final decision is thus made by the operator, who selects the most suitable solution based not only on the quantitative performance of each alternative but also on qualitative insights. This decision is often guided by a set of additional criteria, which may include:

- Risk mitigation: The operator assesses the solutions for their resilience to potential disruptions, such as road closures, traffic delays, or breakdowns, and selects options that offer the best contingency plans.

- Flexibility: Some solutions may provide more flexibility in adjusting routes or reallocating resources, a factor that becomes important when dealing with dynamic changes in customer demand or product availability.

- Stakeholder preferences: The operator may consider customer or client preferences, especially if certain clients have high priority due to contractual obligations or service-level agreements.

- Operator familiarity: The operator's experience with specific routes, vehicles, or clients might influence the final decision, as familiarity with these elements can improve operational efficiency.

- Safety and compliance: Regulatory or safety considerations that are not explicitly modeled but are critical in real-world operations are also weighed in the final decision.

Once the subjective evaluation is complete, the final solution is selected based on the operator's holistic understanding of the situation, balancing both the formal optimization results and the practical realities of transportation. This solution is often a compromise that seeks to meet all efficiency indicators while also ensuring operational reliability under real-world constraints.

Moreover, the operator's feedback is invaluable for improving the overall decision-making framework. If deviations from the projected plan occur during implementation (such as delays due to unforeseen conditions), the lessons learned can be fed back into the system, allowing for continuous improvement of the decision-making process and the models used in earlier approximations. This iterative feedback loop ensures that the system evolves to better handle the dynamic and uncertain nature of transportation operations.

The third solutions approximation with SM provides the final layer of decision-making, where quantitative and qualitative insights converge to produce a solution that is not only optimal in theory but also viable and effective in practice.

3. Case Study and Results

A fuel marketing company, responsible for the supervision and wholesale as well as retail distribution of fuels in a region with 18 customers, was selected as the case scenario. The market segments served by this company include the industrial sector, service center chains, domestic fuels, electricity generation, retail services, deliveries to ships, and the service sector.

For the simulation and application of the proposed method, a general procedure was followed (see Fig. 3). Four members were selected based on their roles and representation of the company's interests. These roles include top management (responsible for decisions related to the company's mission, vision, and strategic objectives), the warehouse functional area (focusing on decisions related to products), the marketing functional area (focused on customer-related decisions), and the transportation functional area (responsible for decisions related to the vehicle fleet).

In the first planning period, gasoline and diesel were selected as the products to be transported, as they were the most in demand during the simulation's time horizon. All products to be transported were compatible with the available fleet for the first transportation period. Since these are unpackaged liquid products, each product is only compatible with itself, ensuring no cross-contamination.

The fleet for the first iteration of transportation planning consists of seven vehicles, each capable of distributing both white and dark products, depending on compatibility. The vehicles have different capacities, with liters chosen as the unit of measurement for capacity. In the economic component, variable cost coefficients play a significant role, primarily depending on fuel consumption rates per kilometer traveled. For the first iteration, it is assumed that all vehicles can serve all customers.

The distances between the depot and the customers, as well as between customers, are provided, along with the respective travel times of the vehicles. The demand and customer service time windows (from 8:00 to 17:00) are known, as

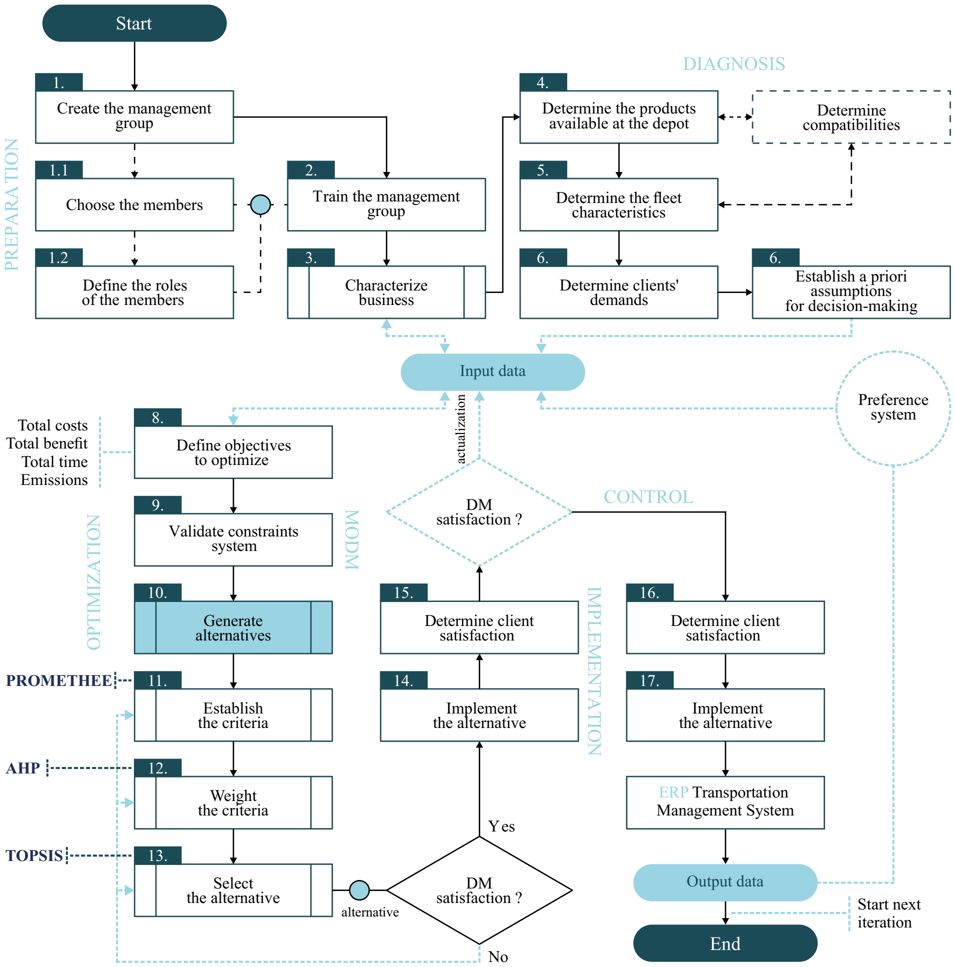


Fig. 3 *Procedure for simulation and application of the multi-objective transportation model.*

is the warehouse's operational time (0:00 to 24:00), which represents the working hours at each node.

During the first iteration, certain decisions were made a priori in the transportation process. All clients were treated with equal importance in the initial approximation. This assumption was made to simplify the modeling process and focus on the primary objectives of minimizing total costs, transportation pollution, and total time. Given that all clients belong to a uniform network of service centers with similar operational roles, this approach allows for a generalized optimization framework applicable across the network. Moreover, it ensures computational feasibility by reducing the complexity of the model. While the equal treatment of clients simplifies the analysis, it also provides a robust foundation for assessing the overall performance of the proposed methodology without introducing additional variability from prioritization schemes. Exploring client-specific priority levels could be considered in future work to capture nuanced logistical dynamics.

The transportation process also utilizes a heterogeneous fleet of vehicles with varying capacities and cost components. These vehicles are required to make multiple trips, depending on customer demand, and must adhere to the time windows established at each node.

3.1 First Solution Approximation: Multi-objective Optimization of the Cargo Transportation

The following objectives were defined for optimization: total costs, total benefits, total time, and total transportation pollution. These objectives were selected to approximate the decision-makers' preferences for logistical decisions. The characteristics and conditions specified in model (19)–(40) were applied, using a heterogeneous fleet of vehicles with varying capacities and both fixed and variable costs. The vehicles were required to make multiple trips depending on customer demand, while adhering to the time windows established at each node.

An initial experimental optimization was conducted with a population size of 30 solutions. However, the results did not meet the conditions necessary to proceed to the second approximation step. Consequently, the population size was increased to 40, which allowed for condition satisfaction.

The experimental implementation and solution generation were carried out using Python on a PC equipped with an Intel® Core™ i5-7200 CPU @ 2.50 GHz (4 CPUs), approximately 2.7 GHz, and 8 GB of RAM. The algorithm was executed 10 times per instance, with the average runtime recorded. The dataset used was based on real data from the unit of analysis. The experiment was also implemented in Go (Golang), to compare the performance. The solution procedure was divided into two approximation steps: generating feasible solutions using the ACO algorithm, followed by selecting the best alternative using MCDM methods. The final population of 40 solutions is illustrated in Fig. 4, representing the outcomes of the iterative optimization process. The results depicted in Fig. 4 illustrate a clear linear relationship between total costs, transportation pollution, and total time, with a small number of outliers. These outliers likely arise from the interaction of heterogeneous fleet constraints, variable customer demands, and the stochastic nature of the ACO algorithm, which explores diverse trade-offs during the iterative process. The limited presence of outliers does not, however, diminish the robustness of the observed trend or the applicability of the proposed methodology.

Each solution represents the route(s) of each vehicle over a day's planning period, based on customer demand and the vehicle's available capacity to serve those customers. The route consists of a sequence of nodes to be visited, starting and ending at the depot. Throughout this process, a total population of 490 solutions was generated.

Since the utility function evaluation involves a non-dimensional transformation of the criteria used in this step, the validation of the pre-condition fulfillment must be carried out in the next, second solutions approximation.

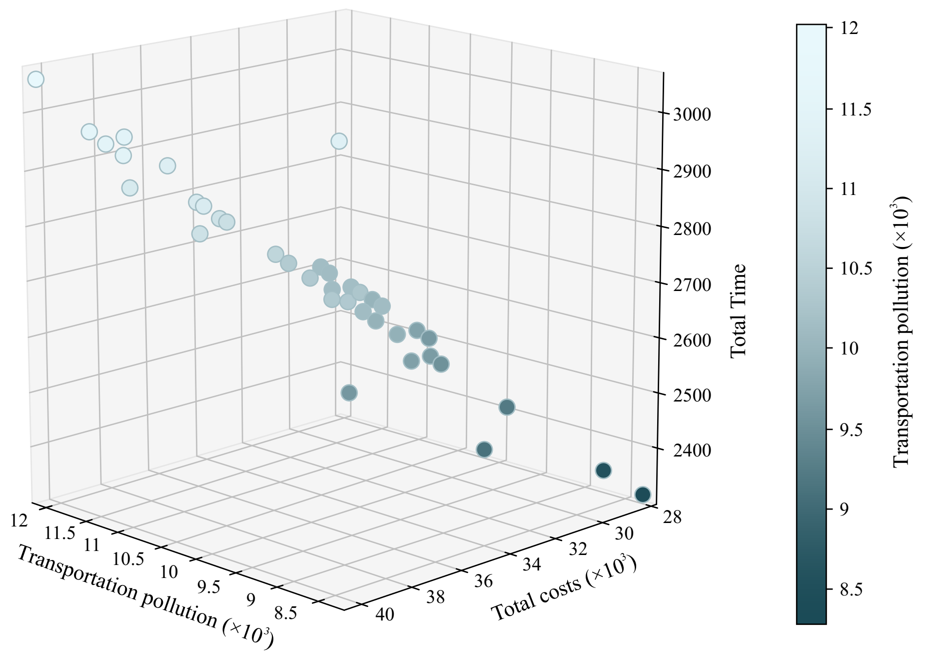


Fig. 4 *Final solutions of the iterative process.*

3.2 Second Solutions Approximation: Selection of Criteria to Evaluate the Alternatives

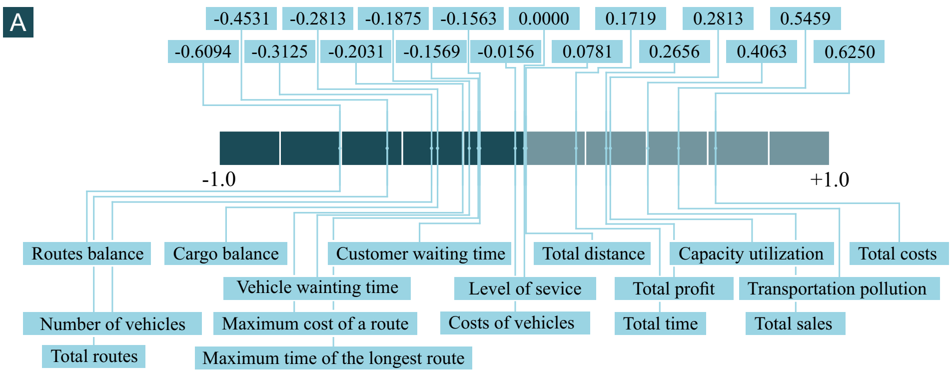
The criteria for evaluating transportation alternatives were defined using the PROMETHEE II method for global ranking. During this phase, the Visual PROMETHEE software was employed, allowing for a more efficient and visual application of the PROMETHEE method. Instead of relying solely on traditional evaluation criteria, the process involved key decision-makers, including senior management, the transportation department, the warehouse department, and the commercial department. These decision-makers contributed to the definition and ranking of the main indicators used to evaluate the solutions generated in the previous stage.

The criteria selected reflect the roles of these decision-makers and include indicators associated with the dimensions of MOT. These indicators were ranked based on their relevance to the decision problem. The main indicators selected were: total costs, total profits, total transportation time, total transportation pollution (emissions), capacity utilization, total sales, total distance, and level of service. Additionally, several sub-criteria were included, such as vehicle costs, maximum route cost, number of vehicles, route balance, total routes, cargo balance, maximum time of the longest route, vehicle waiting time, and customer waiting time.

After determining the relevant criteria, the next step involved establishing the weight of the four decision-makers in the criteria ranking process. Initially, equal weights were assigned to each decision-maker. However, as the procedure progressed, these weights were adapted based on the results. The decision-makers' preferences were collected using a scale (1–5), which enabled an accurate assessment of how they valued each alternative or indicator. Fig. 5A shows the results obtained using interactive tools, where full ranking [1] was applied to illustrate the ranking of alternatives.

To establish the importance of each criterion, the AHP method was employed, where AHP decomposes the problem into a hierarchical structure consisting of a main criterion (the trade-off between all dimensions of the transportation process), sub-criteria (the transportation dimensions), and the alternatives (multi-objective transport indicators). Decision-makers assigned preference values to each alternative in relation to the criteria and sub-criteria using numerical scales. A comparison matrix was constructed to determine the weights of the criteria and sub-criteria, and global priority values for the alternatives were calculated. The final step involved performing a weighted sum of the priorities, resulting in the final classification of alternatives. This procedure facilitated decision-making by considering the preferences and priorities of all decision-makers.

The hierarchical model defined the trade-offs between the dimensions of MOT at the highest level. At the second level, the transportation dimensions were de-



B

		Total costs 1	Transportation pollution 2	Total sales 3	Capacity utilization 4	Total profit 5	Total time 6	Total distance 7	Level of sevice 8	Normalized main eigenvector
Total costs	1	1	5.48	5.48	6.48	3.87	3	5.48	3	33.38%
Transportation pollution	2	0.18	1	0.41	4.47	0.29	0.17	4.47	0.18	6.59%
Total sales	3	0.18	2.45	1	4	4	0.5	5	2	14.07%
Capacity utilization	4	0.15	0.22	0.25	1	2	0.33	4	0.5	5.47%
Total profit	5	0.26	3.46	0.25	0.5	1	0.2	3	0.25	6.25%
Total time	6	0.33	5.92	0.2	3	5	1	4	1	17.80%
Total distance	7	0.18	0.22	0.2	0.25	0.33	0.25	1	0.33	2.70%
Level of sevice	8	0.33	5.48	0.5	2	4	1	3	1	13.73%

Fig. 5 *Full ranking preference analysis processed by PROMETHEE II.*

scribed, and at the third level, the specific indicators related to each dimension were identified. Pairwise comparisons were then collected and recorded in matrices at each hierarchical level, using the Saaty scale to assess the relative importance of the transportation dimensions and their respective indicators.

The normalized decision matrix was constructed based on the preferences provided, and the priority vector for each criterion was calculated by averaging the row values. This vector represents the relative importance of each criterion in the context of the transportation process. This approach provided a structured and weighted evaluation of the criteria, enabling informed decisions to be made based on the priorities of the decision-makers. The priority vector calculation produced the relative weights of each criterion, as shown in Fig. 5B.

To ensure the consistency of the pairwise comparisons, the consistency index and consistency ratio were calculated. With a consistency ratio below 0.1, the level of inconsistency was deemed acceptable. After confirming the consistency of the judgments, the group priority vector was determined by averaging the individual judgment matrices of the decision-makers. This collective judgment matrix was used to recalculate the group priority vector, assigning final judgment weights for the group as a whole. This aggregation of individual priorities into a group priority vector ensured that the final decision reflected the collective perspective of the decision-makers.

Finally, the TOPSIS method was applied to identify the most suitable alternative (route) from a set of options, based on the criteria selected using PROMETHEE and weighted by AHP.

Each alternative was evaluated against the criteria, and the values were normalized to allow comparability. The distances of each alternative from the positive ideal solution (the best) and the negative ideal solution (the worst) were calculated. The alternative closest to the positive ideal solution and furthest from the negative ideal solution was considered the best option.

In constructing the decision matrix, the attributes were transformed into non-dimensional values to allow for comparison. The weighted normalized decision matrix (see Tab. II) was created by incorporating the weights assigned by the decision-makers to each criterion. Each column of the matrix was multiplied by its corresponding weight, introducing the criteria's weighting into the evaluation of the alternatives. The positive ideal solution and the negative ideal solution were identified by finding the maximum values for the benefit criteria and the minimum values for the cost criteria (see Tab. II). These ideal solutions represent the optimal characteristics an alternative should possess for each criterion.

Separation measures were calculated using the Euclidean distance between each alternative and both the positive and negative ideal solutions. The proximity of an alternative to the positive ideal solution was indicated by S_{i+}, while S_{i-} measured the proximity to the negative ideal solution (Columns 10–11, Tab. III). The relative proximity to the ideal solution was then calculated, with values closer to 1 indicating greater similarity to the positive ideal solution (Column 12, Tab. III).

The alternatives were ranked based on their relative proximity to the ideal solution, with the best alternative ranked highest (Column 13, Tab. III).

According to the TOPSIS method, Solution 38 achieved the highest score and ranking, representing the alternative selected by the decision-makers. Information

Solution #	TC	TP	TPr	CU	TS	TT	TD	SL
1	.052	.0102	.022	.009	.010	.028	.004	.022
2	.056	.0110	.022	.009	.010	.030	.005	.022
3	.050	.0099	.022	.009	.010	.027	.004	.022
4	.046	.0113	.022	.009	.010	.030	.005	.022
5	.049	.009	.022	.009	.010	.025	.004	.022
...	...	...	...	...	...	...	...	...
40	.053	.0106	.022	.009	.010	.029	.004	.022
A+	.042	.008	.022	.009	.010	.024	.003	.022
A–	.061	.012	.022	.009	.010	.032	.005	.022

TC – Total costs, TP – Transportation pollution, TPr – Total profit, CU – Capacity utilisation, TS – Total saves, TT – Total time, TD – Total distance, SL – Level of service

Tab. II *Weighed normalized decisions matrix with positive (A+) and negative (A-) ideal solutions by criterion.*

regarding Solution 38's performance, including its fitness in relation to the decision-makers' preferences, is presented in Tab. III. It was confirmed that this solution met the criteria $z_i < z_i^{\sup}$, with $\alpha_i = (z_i^{\sup} - z_i^{inf})\,\forall i = 1, \ldots, r$, and therefore $U(z_1(x), \ldots, z_r(x)) = \min U(z_1(x) + \alpha_1, \ldots, z_r(x) + \alpha_r)$. Consequently, solution 38 (from Tab. III) can be considered the optimal one from the population generated in the first step.

In comparison to the other solutions, Solution 38 demonstrated the shortest travel distance and balanced performance across the fitness functions, validating its selection as the best alternative.

3.3 Third Solutions Approximation: Selection by the System Operator Based on Real Operative Conditions

Real-world operational conditions in transportation systems frequently introduce disruptions that need to be addressed. For instance, after using the TOPSIS method to generate the ordered decision matrix shown in Tab. III and communicating it to the system operator, suppose an accident occurs on the route between nodes 0 and 7.

In this case, the "optimal" Solution 38 must be replaced with another feasible solution. Solutions 23 and 37 also involve traveling on the road from 0 to 7, but Solution 11 avoids this route. Therefore, Solution 11 is selected for implementation.

In this iteration, the system operator's experience in dealing with real-life situations is taken into account. Solution 11, whose details are shown in Tab. III, can be applied because it avoids the problematic route. As a result, the decision adopted by the operator is depicted in Fig. 6.

This solution also satisfies the conditions and can be considered optimal within the solutions population generated in the first approximation step. There are various reasons that may require selecting a solution other than the first one in the

S.	TC	E	TP	CU	TS	TT	TD	SL	S_{i+}	S_{i-}	C_{i+}	R
38	.042	.008	.022	.008	.009	.024	.003	.021	.000	.020	1.00	1
23	.043	.008	.022	.008	.009	.024	.003	.021	.001	.019	.934	2
37	.046	.009	.022	.008	.009	.025	.003	.021	.004	.016	.774	3
11	.048	.009	.022	.008	.009	.026	.004	.021	.007	.013	.664	4
5	.049	.009	.022	.008	.009	.025	.003	.021	.007	.014	.658	5
4	.046	.011	.022	.008	.009	.029	.004	.021	.007	.015	.656	6
35	.049	.009	.022	.008	.009	.026	.004	.021	.007	.013	.647	7
12	.049	.009	.022	.008	.009	.027	.004	.021	.007	.013	.638	8
27	.049	.009	.022	.008	.009	.027	.004	.021	.008	.013	.619	9
6	.050	.009	.022	.008	.009	.026	.004	.021	.008	.012	.594	10
3	.050	.009	.022	.008	.009	.027	.004	.021	.008	.012	.593	11
34	.050	.010	.022	.008	.009	.027	.004	.021	.009	.011	.560	12
28	.050	.010	.022	.008	.009	.027	.004	.021	.009	.011	.558	13
15	.050	.010	.022	.008	.009	.027	.004	.021	.009	.011	.555	14
36	.050	.010	.022	.008	.009	.027	.004	.021	.009	.011	.546	15
18	.051	.010	.022	.008	.009	.027	.004	.021	.009	.011	.536	16
19	.051	.010	.022	.008	.009	.027	.004	.021	.009	.011	.535	17
26	.051	.010	.022	.008	.009	.027	.004	.021	.010	.010	.522	18
24	.051	.010	.022	.008	.009	.027	.004	.021	.010	.010	.511	19
21	.051	.010	.022	.008	.009	.027	.004	.021	.010	.010	.509	20
22	.052	.010	.022	.008	.009	.027	.004	.021	.010	.010	.485	21
1	.052	.010	.022	.008	.009	.027	.004	.021	.010	.010	.478	22
31	.052	.010	.022	.008	.009	.028	.004	.021	.011	.009	.472	23
25	.052	.010	.022	.008	.009	.028	.004	.021	.011	.009	.462	24
30	.052	.010	.022	.008	.009	.028	.004	.021	.011	.009	.448	25
16	.053	.010	.022	.008	.009	.028	.004	.021	.012	.008	.415	26
7	.054	.009	.022	.008	.009	.026	.003	.021	.012	.008	.413	27
40	.053	.010	.022	.008	.009	.028	.004	.021	.012	.008	.392	28
17	.055	.011	.022	.008	.009	.029	.004	.021	.014	.006	.312	29
39	.055	.011	.022	.008	.009	.029	.004	.021	.014	.006	.301	30
13	.056	.011	.022	.008	.009	.029	.004	.021	.015	.005	.275	31
2	.056	.011	.022	.008	.010	.029	.004	.021	.015	.005	.242	32
8	.057	.010	.022	.008	.010	.029	.004	.021	.016	.004	.218	33
9	.057	.011	.022	.008	.010	.030	.004	.021	.016	.004	.214	34
29	.058	.011	.022	.008	.010	.030	.004	.021	.017	.003	.146	35
33	.058	.011	.022	.008	.010	.030	.004	.021	.018	.003	.141	36
20	.059	.011	.022	.008	.010	.030	.004	.021	.018	.002	.118	37
10	.059	.011	.022	.008	.010	.030	.004	.021	.018	.002	.115	38
14	.059	.011	.022	.008	.010	.030	.004	.021	.019	.001	.091	39
32	.061	.012	.022	.008	.010	.031	.005	.021	.020	.000	.000	40
z_i^{sup}	.061	.012	.022	.008	.010	.031	.005	.021	–	–	–	–

S. – Solution, TC – Total costs, TP – Transportation pollution, TPr – Total profit, CU – Capacity utilisation, TS – Total saves, TT – Total time, TD – Total Distance, SL – Level of service, R – Rank

Tab. III *Decisions matrix based on the technique for order of preference by similarity to ideal solution (TOPSIS).*

decision matrix. For example, the daily transportation plan might need to be altered due to vehicle breakdowns or other unforeseen circumstances.

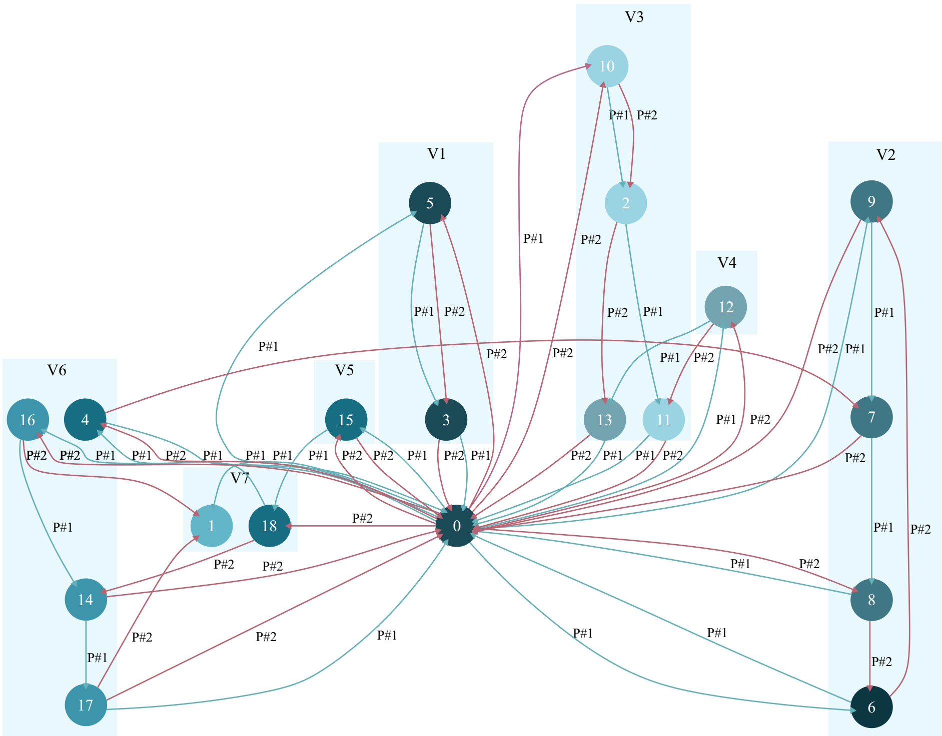


Fig. 6 *Graph of the solution 11.*

4. Discussion

The intricate landscape of transportation logistics and VRPs has grown increasingly important in the context of global industrialization and trade. This research underscores the multifaceted nature of VRPs and emphasizes the need for a robust MOT model that effectively integrates decision-makers' preferences with mathematical formulations and hybrid optimization strategies.

This study's exploration of various optimization dimensions—economic, spatial, temporal, and environmental—aligns closely with previous research findings. The economic dimension, as highlighted by Li et al. [26], X. Wang et al. [41], Yin [43], Zarouk et al. [44], and Zhou and Zhao [49], includes critical factors such as total costs, driver remuneration, and overall profitability, all of which are essential for evaluating the financial viability of transportation strategies. The spatial dimension, which focuses on metrics such as distance traveled and vehicle utilization, resonates with the observations of Molano et al. [31], H. Wang et al. [39], and W. Zhang et al. [48], who emphasize the importance of optimizing route efficiency to improve service levels.

The temporal dimension, centered on customer satisfaction and service levels, has become increasingly significant as organizations strive to meet the demands of a fast-paced and dynamic marketplace. Studies by Ghannadpour et al. [13], Zhang and Wang [46], and Chavez et al. [6] underscore the importance of incorporating time constraints into routing decisions to enhance service delivery and customer engagement. This reinforces the need for decision-making frameworks that account for temporal variables.

Environmental considerations also play a crucial role in transportation optimization. This study highlights the limitations of existing literature, particularly regarding emissions modeling beyond CO2. While focusing on total emissions is a valuable step, as noted by Dutta et al. [9], H. Wang et al [39], and Yin [43], future research should strive to encompass a broader range of pollutants and their impacts on sustainability. Expanding the environmental dimension would offer a more nuanced understanding of transportation's ecological footprint.

The review of methodologies used to address multi-objective vehicle routing problems reveals a variety of approaches. Although many studies have focused on deterministic models, a shift toward stochastic frameworks is becoming more prevalent, as seen in the works of Tan et al. [36], Ghannadpour et al. [13], Gee et al. [12], and Men et al. [28]. This shift is critical for addressing the uncertainties that arise in real-world transportation scenarios, thereby improving the robustness of proposed solutions.

Despite advancements in solution methodologies, a significant gap remains in the systematic incorporation of decision-maker preferences within the optimization process. The exclusion of subjective criteria from multi-objective frameworks limits their practical applicability and contradicts the essence of multi-objective optimization. Future research should focus on developing frameworks that integrate preference structures effectively, ensuring that the outcomes align with the strategic objectives of stakeholders.

The study also highlights the importance of hybrid optimization strategies, as demonstrated in the works of J. Wang et al. [40], Dutta et al. [9], H. Zhang et al. [45], and W. Zhang et al. [48]. Hybrid strategies combine the strengths of various algorithms to address the complexities of multi-objective decision-making. This approach not only enhances the robustness of solutions but also provides a more tailored response to the specific characteristics of each routing problem.

While substantial progress has been made in modeling and solving multi-objective vehicle routing problems, challenges remain in developing universally applicable frameworks. This research advances transportation optimization methodologies by deepening the understanding of the interplay between mathematical modeling, solution strategies, and decision-maker preferences.

The proposed MOT model offers substantial strengths, particularly in optimizing complex vehicle routing problems while accommodating real-world constraints. The integration of approximate-combinatorial methods and hybrid optimization techniques, such as ant colony optimization and evolutionary algorithms, ensures robust and efficient handling of the combinatorial complexity inherent in VRPs. This is particularly crucial for scenarios involving spatial, temporal, and capacity constraints, as demonstrated in the case study focusing on fuel distribution.

One of the key strengths of the model is its ability to integrate decision-makers' preferences directly into the optimization framework. By employing MCDM methods, including PROMETHEE II, AHP, and TOPSIS, the model not only generates multiple feasible solutions but also refines them to align with organizational priorities and objectives. This enables decision-makers to achieve a balanced trade-off among competing objectives, such as minimizing costs, transportation time, and environmental impact while maximizing service levels and operational efficiency.

The robustness of the model is further underscored by its scalability and adaptability to various operational contexts. The case study involving a heterogeneous fleet and 18 customers illustrates its practical application in addressing real-world logistics challenges. The optimization results highlight the model's capacity to significantly reduce transportation costs and emissions while maintaining high service quality. Notably, Solution 38, identified as the optimal alternative using the TOPSIS method, demonstrated the model's effectiveness in balancing multiple conflicting objectives.

Additionally, the model's ability to integrate subjective decision-making factors into the optimization process enhances its resilience to dynamic and uncertain operational conditions. By allowing system operators to incorporate their expertise and situational awareness, the model bridges the gap between theoretical optimization and practical decision-making. This was evident in the scenario where real-world disruptions, such as an accident on a planned route, necessitated the selection of an alternative solution. The model's flexibility to adapt to such unforeseen events underscores its practicality and reliability in operational settings.

The proposed model addresses theoretical complexities while showing promising potential for practical application. By incorporating decision-makers' preferences and employing hybrid optimization techniques, it provides a structured approach to managing complex transportation logistics. For instance, the case study on fuel distribution illustrates its capability to optimize routes for a heterogeneous fleet, contributing to improvements in cost-efficiency, operational effectiveness, and environmental sustainability. These initial findings suggest that the model could be valuable in various areas of transportation logistics, including supply chain management and urban delivery systems.

The model's ability to combine qualitative insights with quantitative optimization also supports its adaptability to operational uncertainties. For example, its flexibility in responding to dynamic conditions, such as route disruptions or fluctuating demand, enhances its practical utility. While these attributes are promising, further research and testing across diverse scenarios are necessary to confirm the model's broader applicability and robustness. Exploring advanced predictive analytics could also enhance its ability to anticipate and respond to emerging challenges in transportation systems.

This study provides an initial step toward integrating theoretical optimization approaches with practical decision-making in transportation logistics. While the findings are encouraging, additional work is needed to fully realize the model's potential and to refine its application in diverse and complex operational contexts.

5. Conclusion

The findings of this research demonstrate the potential to optimize decision-making under multiple criteria by incorporating MCDM procedures as models of decision-makers' subjective evaluations in complex systems. The approximate-combinatorial method provides a viable approach for optimizing utility functions through successive approximations.

Decision-makers require flexibility in transitioning between structured MODO models and MCDM models to achieve quantitative precision when handling continuous and discrete challenges. Simultaneously, the SM adds a crucial qualitative layer that captures the human-centric aspects of decision-making, accounting for real-world complexities that structured models may overlook.

In this context, the proposed model for optimizing the operation of transportation systems has proven to be an effective tool for informed and balanced decision-making. The integration of hybrid optimization methods enhances both flexibility and speed, underscoring the importance of adopting advanced decision-making approaches in complex business environments. This approach—combining multi-objective and multi-criteria methods—not only offers a competitive advantage but also provides valuable insights for companies facing similar challenges in managing their transportation processes.

By demonstrating how mathematical models, decision-making preferences, and hybrid optimization methods can be effectively integrated, this research contributes to the advancement of transportation logistics. Future studies should focus on refining these methodologies while addressing the practical challenges faced by decision-makers in dynamic, real-world environments. The continued development of these approaches will ensure that companies can meet their operational goals while balancing efficiency, sustainability, and customer satisfaction.

Abbreviations

ACO	Ant colony optimization
AHP	Analytic hierarchy process
CO2	Carbon dioxide (used for emissions measurement)
MCDM	Multi-criteria decision-making
MODO	Multi-objective decision optimization
MOT	Multi-objective transportation
MoVRP	Multi-objective VRP
NP	Nondeterministic polynomial time
PROMETHEE	Preference ranking organization method for enrichment evaluation
SL	Service level
SM	Subjective models
TC	Total cost
TP	Transportation pollution
TPr	Total profit loss of transportation
TD	Total distance

TOPSIS	Technique for order preference by similarity to ideal solution
TS	Total sales
TT	Total transportation time
VRP	Vehicle routing problem

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