QUANTUM MULTIDIMENSIONAL MODELS OF COMPLEX SYSTEMS

*M. Svítek**

Abstract: The paper presents a new methodology how to extend the well-known quantum model [2] with $(2N-1)$ free parameters (moduli and phases) of wave probabilistic functions $\psi(A_i)$ assigned into events A_i, $i \in \{1, 2, \ldots, N\}$ to $\frac{N\cdot(N+1)}{2}$ free parameters necessary for full N-dimensional representation of complex system. Our approach generally enables to include additional functions applied on events A_i, $i \in \{1, 2, \ldots, N\}$. In the paper, we will demonstrate this mathematical instrument on additional wave probabilistic functions $\psi(A_k \cap A_m \cap \cdots \cap A_n)$ connected with macroscopic events' intersections $A_k \cap A_m \cap \cdots \cap A_n$ where $k, m, \ldots, n \in \{1, 2, \ldots, N\}$.

Key words: *quantum system theory, quantum informatics, complex systems, information physics, extended quantum models, wave probabilistic functions, smart systems*

Received: March 19, 2019 **DOI:** 10.14311/NNW.2019.29.022
Revised and accepted: October 27, 2019

1. Introduction

A probability space consists of a sample space S and a probability function $P(\cdot)$, mapping the events of S to real numbers in $[0, 1]$, such that $P(S) = 1$, and if $A_1, A_2, \ldots, A_N$ is a sequence of disjoint events, then the sum rule is fulfilled:

$$P\left(\bigcup_{i \in N} A_i\right) = \sum_{i \in N} P(A_i). \tag{1}$$

If the events A_1, $A_2, \ldots, A_N$ are not disjoint, the following (intersection and union) rules can be used:

$$\begin{aligned} &P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot \\ &\cdot P(A_3|A_1 \cap A_2) \ldots P(A_N|A_1 \cap \cdots \cap A_{N-1}), \end{aligned} \tag{2}$$

$$\begin{aligned} & P(A_1 \cup A_2 \cup \cdots \cup A_N) = \\ = \ & \sum_{i-1}^{N} P(A_i) - \sum_{i<j}^{N} P(A_i \cap A_j) + \sum_{i<j<k}^{N} P(A_i \cap A_j \cap A_k) + \ldots \\ + \ & (-1)^{N-1} \cdot P(A \cap A \cap \cdots \cap A_N). \end{aligned} \tag{3}$$

*Miroslav Svítek; Faculty of Transportation Sciences, CTU Prague, E-mail: svitek@fd.cvut.cz

Considering the basic laws of probability Eq. (3), we need generally $\frac{N\cdot(N+1)}{2}$ free parameters.

The main goal of the paper is to extend a quantum model [2] with $(2N-1)$ free parameters in such a way that it could describe the system with more parameters. In this way, the curse of dimensionality can be overcome, at least in part, with the increasing dimensionality of the system, the number of required parameters for its full description rapidly increases. The Section 2 summarizes the basic definition and some features of quantum models. Section 3 shows the applicability limits of current quantum models. In Section 4, the new approach to quantum extended models is described together with illustrative examples presented in Section 5. Section 6 concludes the paper.

2. Quantum models

Let us suppose N events A_i, $i \in \{1,2,\ldots,N\}$ with defined probabilities $P(A_i)$, $i \in \{1,2,\ldots,N\}$, and N wave probabilistic functions:

$$\psi(A_i) = \alpha_i \cdot e^{j\cdot\upsilon_i} = \sqrt{P(A)_i}\cdot e^{j\cdot\upsilon i}, i \in \{1,2,\ldots,N\}, \tag{4}$$

together with their superposition state $|\psi\rangle$ as a quantum object [3]:

$$|\psi\rangle = \psi(A_1)\cdot|A_1\rangle + \psi(A_2)\cdot|A_2\rangle + \cdots + \psi(A_N)\cdot|A_N\rangle, \tag{5}$$

with moduli $\sqrt{P(A_i)}$ and phases υ_i, where the reference phase assigned to event A_1 is chosen as $\upsilon_1 = 0$. The intersection and union rules for quantum models were defined in [4]:

$$P(|A_1\rangle \cup |A_2\rangle \cup \cdots \cup |A_N\rangle) = \left|\sum_{i=1}^{N}\psi(A_i)\right|^2, \tag{6}$$

$$P(|A_r\rangle \cap |A_s\rangle) = \lim_{\sqrt{\substack{P(A_k)\\ k\neq r,s}}\to 0}\left[\psi^*(A_r)\cdot\psi(A_s) + \psi(A_r)\cdot\psi^*(A_s)\right], \tag{7}$$

where symbol ψ^* expresses a complex conjugate of ψ.

Quantum model Eq. (4) and (5) provides only $(2N-1)$ parameters – moduli $|\psi(A_i)|$ and phases ν_i of wave probabilistic functions $\psi(A_i) = |\psi(A_i)|\cdot e^{j\cdot\nu i}$. Let us show the dimension limit of quantum model on following illustrative example.

Example 1 – N-dimensional distribution and its approximation by quantum model We will assume N-dimensional distribution for which we need to specify all the values of the $N \times N$ covariance matrix $\sigma_{i,j}$. Due to symmetry, we need only $\frac{N\cdot(N+1)}{2}$ parameters.

Since we cannot determine the covariance matrix $\sigma_{i,j}$ exactly, we therefore need to come up with an approximate description, a description that would require fewer parameters.

Instead of representing each quantity δ_i as an N-dimensional vector $a_i = (a_{i,1},\ldots,a_{i,N})$ corresponding to $\delta_i = \sum_{j=1}^{N} a_{ij}\cdot X_j$ where $\{X_1,\ldots, X_N\}$ are independent standard random variables, we select some value $k << N$ and represent each quantity δ_i as a k-dimensional vector corresponding to $\delta_i = \sum_{j=1}^{k} a_{ij}\cdot X_j$. For $k = 2$, the approximation leads to a Quantum model [1].

3. Practical applications of quantum models

In quantum (wave) approximation, only $(N-1)$ phase parameters among N-dimensional events A_i, $i \in \{1, 2, \ldots, N\}$ are available. For simplicity, we will not be concerned by moduli $|\psi(A_i)|$ because they have no impact on phase parameters. Look at practical examples of quantum models' applicability.

3.1 Ordering models

Suppose the unique ordering of the events $\{A_1, A_2, \ldots, A_N\}$ where each index represents the event's order in sequence and the distance between two events l, m is defined as $d_{l,m} = |l-m|$. In quantum notation, the phase difference $\nu_{l,m} = \nu_l - \nu_m$ represents correlation (link) between two events. The quantum (wave) model is fully applicable in case the correlation between events l, m is dependent only on the distance between them $d_{l,m} = |l-m|$ and not on their position. For this example, the new phase parameters $\tilde{\nu}_i$ could be introduced:

$$\begin{aligned}
\tilde{\nu}_1 &= \nu_2 - \nu_1 = \nu_3 - \nu_2 = \ldots \nu_{N-1} - \nu_{N-2} = \nu_N - \nu_{N-1} \\
\tilde{\nu}_2 &= \nu_3 - \nu_1 = \nu_4 - \nu_2 = \cdots = \nu_N - \nu_{N-2} \\
&\vdots \\
\tilde{\nu}_{N-1} &= \nu_n - \nu_1.
\end{aligned} \tag{8}$$

From (8) it is clear that we can describe whole system by $(N-1)$ phase parameters $(\tilde{\nu}_1, \tilde{\nu}_2, \ldots, \tilde{\nu}_{N-1})$. This model is typical for time-invariant subsystems [5] where the correlation is dependent only on the time differences between two realizations.

3.2 Incremental models

Let us suppose the existence of a reference event (a phase of event A_1) typically equal to $\nu_1 = 0$, from which we measure the correlations (links) to other events. In incremental model, due to additivity all other correlations could be computed from phases. For example, a phase difference between events k and $(k+d)$ has to be equal to $\nu_d = \nu_{k+d} - \nu_k$. This situation is standard for quantum mechanics [7] where the reference represents a zero energy and other energy levels are gradually increased by step functions. Such system is represented by following phase structure as:

$$\begin{aligned}
\tilde{\nu}_1 &= \nu_1 \\
\tilde{\nu}_2 &= \nu_1 + \nu_2 \\
&\vdots \\
\tilde{\nu}_N &= \nu_1 + \nu_2 + \cdots + \nu_N.
\end{aligned} \tag{9}$$

This model corresponds to gradual evolution of complex systems [3, 4]. At the beginning we have only subsystem S_1. After adding the subsystem S_2 the correlation yields into encapsulation into the new subsystem $S_{1,2}$. Now we can imagine adding the subsystem S_3 to subsystem $S_{1,2}$, which plays the role of a reference for subsystem S_3. We can continue up to subsystem S_N that will be dependent on previous encapsulated subsystem $S_{1,2,\ldots,N-1}$.

4. Quantum extended models

The main idea of extended quantum model is to include into quantum superposition not only events itself A_i, $i \in \{1, 2, \ldots, N\}$ but also n additional events' functions, e.g. $f_k(A_1, \ldots, A_N), k \in \{i, \ldots, n\}$. Then the modified quantum model can be written:

$$\begin{aligned}\psi(A) &= \psi(A_1) \cdot |A_1\rangle + \psi(A_2) \cdot |A_2\rangle + \cdots + \\ &+ \psi(A_N) \cdot |A_N\rangle + \psi(f_1(A_1, \ldots, A_N)) \cdot |f_1(A_1, \ldots, A_N)\rangle + \cdots + \\ &+ \psi(f_n(A_1, \ldots, A_N)) \cdot |f_n(A_1, \ldots, A_N)\rangle. \end{aligned} \tag{10}$$

Such approach brings many possibilities how rapidly extend the dimensionality of complex system.

Example 2 – dimensionality analyze of binary functions In case of N binary events $A \in \{0, 1\}$, $i \in \{1, 2, \ldots, N\}$ we have 2^N different variants of outputs combinations. If we suppose a binary function applied on each events' combination, theoretically we can achieve 2^{2^N} different variants of functions outputs.

Let us suppose an example with $N = 2$ that means 4 combinations of binary events A_1, $A_2 \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. By application of two dimensional function $f(A_1, A_2)$ we can return $4^2 = 16$ variants of different outputs. For $N = 3$ we have 8 binary combinations of events A_1, A_2, A_3 but 256 possible variants of binary functions. This example demonstrates how fast the number of free parameters increases by adding additional binary functions.

4.1 Extended intersectional quantum models

For practical feasibility, we will restrict us only to events' intersections $f_h(A_1, \ldots, A_N) = A_k \cap A_m \cap \cdots \cap A_r$, $k, m, \ldots, r \in \{1, 2, \ldots, N\}$. The extended quantum model can be than rewritten:

$$\begin{aligned}\psi(A) &= \psi(A_1) \cdot |A_1\rangle + \psi(A_2) \cdot |A_2\rangle + \cdots + \\ &+ \psi(A_N) \cdot |A_N\rangle + \psi(A_1 \cap A_2) \cdot |A_1 \cap A_2\rangle + \cdots + \\ &+ \psi(A_k \cap A_m \cap \cdots \cap A_r) \cdot |A_k \cap A_m \cap \cdots \cap A_r\rangle. \end{aligned} \tag{11}$$

There are of course other possibilities how to include additional information into quantum model but the intersections seem to be more natural and easily applicable. In this case, it is possible to manipulate with different combinations of events' intersections ($\otimes$ is Kronecker product [5]):

$$\begin{aligned}|A_i\rangle \otimes |A_i \cap A_j\rangle &\Rightarrow |A_i \cap A_j\rangle \\ |A_i \cap A_k \cap A_r\rangle \otimes |A_k \cap A_r\rangle &\Rightarrow |A_i \cap A_k \cap A_r\rangle \\ |A_i \cap A_k \cap A_r\rangle \otimes |A_p \cap A_q\rangle &\Rightarrow |A_i \cap A_k \cap A_r \cap A_p \cap A_q\rangle. \end{aligned} \tag{12}$$

Such logical rules give us mathematical instrument for wave probabilistic interferences that yields to modelling of new multi-dimensional complex systems like quantum entanglement [2].

4.2 Interpretation of extended intersectional quantum models

A superposition of different events together with some events' intersections yields into the extended intersectional quantum model. We can divide events' intersections into two groups: inner (microscopic) and outer (macroscopic).

Inner (microscopic) intersections represent emergent feature of complex system and are modeled by phase differences of wave probabilistic functions [2]. Due to positive or negative sigs, they could go to either inner attraction or repelling of events. These features yields into well-known quantum modelling [5, 9].

Outer (macroscopic) intersections could be seen as the additional observable behavior that could be considered as the new event (quantity) of studied system. Because of macroscopic nature, we need use only classical probability theory that was developed for description of macroscopic phenomena.

For example, the extended quantum model enables modelling links between two different macroscopic intersections through their wave probabilistic phases. Entanglement than can be realized not only among pure events but also among events' intersections.

Example 3 – social model of relation among company employees We can suppose to have N employees A_i, $i \in \{1, 2, \ldots, N\}$ with the inner links that are defined psychologically, ability to take responsibility, etc. Such characteristic can be scarcely measured. Its observations are limited to inner phase parameters that are extracted only from holistic behavior of the team.

On the other hand, there are outer (macroscopic) links that have strong impacts on holistic system' behavior and are easily identifiable. We can state e.g. family relationships, schoolmates, etc. Such macroscopic links should be taken into our model to catch better details. If I am employee A_r I am influenced by all other employees A_k, $k \neq r$ and also by links to additional (macroscopic) employees groups, e.g. $A_k \cap A_c \cap A_d$, $A_q \cap A_l \cap A_t \cap A_o$. Taking into consideration the all relations among studied group of employees the holistic social model can be better specified.

5. Application of extended intersectional quantum models

In many practical applications of quantum models, there is a demand for description of complex networks [8–10]. Optimal management of complex systems consists of the best arrangement of all network nodes represented by amplitudes and phases of all components [2]. The applicability of presented approach will be shown on following examples.

Example 4 – incremental/ordering quantum model We can assume events A_1, A_2, A_3, A_4 represented by probabilities $P(A_1), P(A_2), P(A_3), P(A_4)$ with assigned wave probabilistic functions:

$$\psi(A_1) = \sqrt{P(A_1)} \cdot e^{j \cdot \varphi_1}, \psi(A_2) = \sqrt{P(A_2)} \cdot e^{j \cdot \varphi 2}, \tag{13}$$

$$\psi(A_3) = \sqrt{P(A_3)} \cdot e^{j \cdot \varphi_3}, \psi(A_4) = \sqrt{P(A_4)} \cdot e^{j \cdot \varphi_4}. \tag{14}$$

The union of all events Eq. (6) can be given as follows:

$$\begin{aligned} & P(A_1 \cup A_2 \cup A_3 \cup A_4) = |\psi(A_1) + \psi(A_2) + \psi(A_3) + \psi(A_4)|^2 = \\ = \ & P(A_1) + P(A_2) + P(A_3) + P(A_4) + \\ + \ & 2 \cdot \sqrt{P(A_1) \cdot P(A_2)} \cdot \cos(\varphi_2 - \varphi_1) + 2 \cdot \sqrt{P(A_1) \cdot P(A_3)} \cdot \cos(\varphi_3 - \varphi_1) + \\ + \ & 2 \cdot \sqrt{P(A_1) \cdot P(A_4)} \cdot \cos(\varphi_4 - \varphi_1) + 2 \cdot \sqrt{P(A_2) \cdot P(A_3)} \cdot \cos(\varphi_3 - \varphi_2) + \\ + \ & 2 \cdot \sqrt{P(A_2) \cdot P(A_4)} \cdot \cos(\varphi_4 - \varphi_2) + 2 \cdot \sqrt{P(A_3) \cdot P(A_4)} \cdot \cos(\varphi_4 - \varphi_3). \end{aligned} \tag{15}$$

Comparing with classical probabilistic rule Eq. (3), we can extract:

$$\begin{aligned} P(A_1 \cap A_2) &= 2 \cdot \sqrt{P(A_1 \cdot A_2)} \cdot \cos(\varphi_2 - \varphi_1), \\ P(A_1 \cap A_3) &= 2 \cdot \sqrt{P(A_1 \cdot A_3)} \cdot \cos(\varphi_3 - \varphi_1), \\ P(A_1 \cap A_4) &= 2 \cdot \sqrt{P(A_1 \cdot A_4)} \cdot \cos(\varphi_4 - \varphi_1), \\ P(A_2 \cap A_3) &= 2 \cdot \sqrt{P(A_2 \cdot A_3)} \cdot \cos(\varphi_3 - \varphi_2), \\ P(A_2 \cap A_4) &= 2 \cdot \sqrt{P(A_2 \cdot A_4)} \cdot \cos(\varphi_4 - \varphi_2), \\ P(A_3 \cap A_4) &= 2 \cdot \sqrt{P(A_3 \cdot A_4)} \cdot \cos(\varphi_4 - \varphi_3). \end{aligned} \tag{16}$$

To obtain the ordering quantum model (Section 3.1) we provide transformation and compute following phases:

$$\tilde{\nu}_1 = \varphi_2 - \varphi_1 = \varphi_3 - \varphi_2 = \varphi_4 - \varphi_3, \tag{17}$$

$$\tilde{\nu}_2 = \varphi_3 - \varphi_1 = \varphi_4 - \varphi_2, \tag{18}$$

$$\tilde{\nu}_3 = \varphi_4 - \varphi_1. \tag{19}$$

The phases $\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3$ fully describe the quantum ordering model and there is not necessary to provide any approximation.

Example 5 – extended intersectional quantum model Let us use previous example and to add into this model one additional macroscopic intersection $P(A_3 \cap A_4)$ represented by wave probabilistic function:

$$\psi(A_3 \cap A_4) = \sqrt{P(A_3 \cap A_4)} \cdot e^{j \cdot \nu_{3,4}}. \tag{20}$$

The extended intersectional quantum model can be written in "bra-ket" notation as:

$$\begin{aligned} \psi(A_1, A_2, A_3, A_4) = \psi(A_1) \cdot |A_1\rangle + \psi(A_2) \cdot |A_2\rangle + \\ + \psi(A_3) \cdot |A_3\rangle + \psi(A_4) \cdot |A_4\rangle + \psi(A_3 \cap A_4) \cdot |A_3 \cap A_4\rangle. \end{aligned} \tag{21}$$

The union of all events Eq. (15) can be enlarged:

$$\begin{aligned} & P(A_1 \cup A_2 \cup A_3 \cup A_4) = \psi(A_1, A_2, A_3, A_4) \cdot \psi^*(A_1, A_2, A_3, A_4) = \\ = \ & [\psi(A_1) \cdot |A_1\rangle + \psi(A_2) \cdot |A_2\rangle + \psi(A_3) \cdot |A_3\rangle + \psi(A_4) \cdot |A_4\rangle + \\ & + \psi(A_3 \cap A_4) \cdot |A_3 \cap A_4\rangle] \cdot [\psi^*(A_1) \cdot |A_1\rangle^* + \psi^*(A_2) \cdot |A_2\rangle^* + \\ & + \psi^*(A_3) \cdot |A_3\rangle^* + \psi^*(A_4) \cdot |A_4\rangle^* + \psi^*(A_3 \cap A_4) \cdot |A_3 \cap A_4\rangle^*]. \end{aligned} \tag{22}$$

We can use the following "composition" rules:

$$\begin{aligned} & \psi(A_i)\cdot|A_i\rangle\cdot\psi^*(A_j)\cdot|A_j\rangle^* + \\ + \quad & \psi(A_j)\cdot|A_j\rangle\cdot\psi^*(A_i)\cdot|A_i\rangle^* = \\ = \quad & \sqrt{P(A_i)\cdot P(A_j)}\cdot\cos(\varphi_i-\varphi_j)\cdot|A_i\cap A_j\rangle, \end{aligned} \tag{23}$$

$$\begin{aligned} & \psi(A_i)\cdot|A_i\rangle\cdot\psi^*(A_j\cap A_k)\cdot|A_j\cap A_k\rangle^* + \\ + \quad & \psi(A_j\cap A_k)\cdot|A_j\cap A_k\rangle\cdot\psi^*(A_i)\cdot|A_i\rangle^* = \\ = \quad & \sqrt{P(A_i)\cdot P(A_j\cap A_k)}\cdot\cos(\varphi_i-\varphi_{j,k})\cdot|A_i\cap A_j\cap A_k\rangle, \end{aligned} \tag{24}$$

$$\begin{aligned} & \psi(A_i)\cdot|A_i\rangle\cdot\psi^*(A_i\cap A_k)\cdot|A_i\cap A_k\rangle^* + \\ + \quad & \psi(A_i\cap A_k)\cdot|A_i\cap A_k\rangle\cdot\psi^*(A_i)\cdot|A_i\rangle^* = \\ = \quad & \sqrt{P(A_i)\cdot P(A_i\cap A_k)}\cdot\cos(\varphi_i-\varphi_{i,k})\cdot|A_i\cap A_k\rangle. \end{aligned} \tag{25}$$

The probabilistic union of enlarged intersectional model can be computed:

$$\begin{aligned} P(A_1\cup A_2\cup A_3\cup A_4) &= |\psi(A_1)+\psi(A_2)+\psi(A_3)+\psi(A_4)+\psi(A_3\cap A_4)|^2 = \\ &= P(A_1)+P(A_2)+P(A_3)+P(A_4)+P(A_3\cap A_4) + \\ &+ 2\cdot\sqrt{P(A_1)\cdot P(A_2)}\cdot\cos(\varphi_2-\varphi_1) + \\ &+ 2\cdot\sqrt{P(A_1)\cdot P(A_3)}\cdot\cos(\varphi_3-\varphi_1) + \\ &+ 2\cdot\sqrt{P(A_1)\cdot P(A_4)}\cdot\cos(\varphi_4-\varphi_1) + \\ &+ 2\cdot\sqrt{P(A_2)\cdot P(A_3)}\cdot\cos(\varphi_3-\varphi_2) + \\ &+ 2\cdot\sqrt{P(A_2)\cdot P(A_4)}\cdot\cos(\varphi_4-\varphi_2) + \\ &+ 2\cdot\sqrt{P(A_3)\cdot P(A_4)}\cdot\cos(\varphi_4-\varphi_3) + \\ &+ 2\cdot\sqrt{P(A_1)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_1) + \\ &+ 2\cdot\sqrt{P(A_2)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_2) + \\ &+ 2\cdot\sqrt{P(A_3)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_3) + \\ &+ 2\cdot\sqrt{P(A_4)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_4). \end{aligned} \tag{26}$$

In addition of intersections Eq. (16), $P(A_3\cap A_4)$ was extended to:

$$\begin{aligned} \tilde{P}(A_3\cap A_4) = P(A_3\cap A_4) + 2\cdot\sqrt{P(A_3)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_3) + \\ +2\cdot\sqrt{P(A_4)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_4), \end{aligned} \tag{27}$$

where $\tilde{P}(A_3\cap A_4)$ is modified extended intersection probability $P(A_3\cap A_4)$ enriched with inner links to events A_3, A_4.

The third order intersections appeared due to added wave function Eq. (20):

$$\begin{aligned} P(A_1\cap A_3\cap A_4) = 2\cdot\sqrt{P(A_1)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_1), \\ P(A_2\cap A_3\cap A_4) = 2\cdot\sqrt{P(A_2)\cdot P(A_3\cap A_4)}\cdot\cos(\varphi_{3,4}-\varphi_2). \end{aligned} \tag{28}$$

In a similar way, more variants of macroscopic intersections can be included into extended intersectional quantum model. The more intersections (pieces of information), the more available additional parameters and the more possibility to model multi-dimensional complex system.

6. Conclusion

The main goal of the paper was to find the way to overcome the dimensionality problem of quantum models. The proposed solution supposes including events' combinations (function of events) into mass parallel quantum model as the additional macroscopic state. For better feasibility, we concentrated only on the intersectional macroscopic models (intersections of selected events). The events' intersections could be easily interpreted and logical rules, how to work with them, were developed.

For complete description of all correlations (links) among events A_i, $i \in \{1, 2, \dots, N\}$ we need to specify at least $\frac{N\cdot(N+1)}{2}$ free parameters. This corresponds to requirements how many different intersections we need to include into the extended intersectional quantum model. We revealed on illustrative example how to incorporate new information into the extended model. The more intersections the better model we can build.

The benefit of the extended model is that a well-known entanglement cannot be only among the pure events but also among the different intersections or among the combinations of intersections and pure events. These features bring new possibilities for modelling especially soft systems with enormous links and interconnections.

In quantum physic, moduli represent typically energy [7]. In our approach, the mathematical instrument of wave probabilities has, therefore, much broader applications than in physics [12, 13]. In system sciences, for example, we can evaluate by quantum models other system features like the ability to create alliances [8], the ability for adaptation (the quickest response to changes) [6], etc. The presented methodology can be further enlarged not only to entanglement among pure events and events' functions, but also it can add an enlargement of different system processes and their functions. If we imagine the complexity of such a model [11], we are coming closer to Kaufmann tissue [6].

Acknowledgement

This work was supported by the Project AI & Reasoning CZ.02.1.01/0.0/0.0/15_003/0000466 and the European Regional Development Fund.

References

[1] SVÍTEK M., KOSHELEVA O., KREINOVICH V., NGUYEN T. N. Why Quantum (Wave Probability) Models Are a Good Description of Many Non-quantum Complex Systems, and How to Go Beyond Quantum Models), pp. 168–175, In: Kreinovich et al. (Eds.): *Beyond Traditional Probabilistic Methods in Economics, Springer Nature Switzerland AG 2019*, SCI 809, 2019, doi: 10.1007/978-3-030-04200-4_13.

[2] SVÍTEK M. Towards complex system theory, Tutorial, *Neural Network World,* 2015, 25(1), pp. 5–33, ISSN 1210-0552.

[3] SVÍTEK M.: Applying Wave Probabilistic Functions for Dynamic System Modeling, *IEEE transactions on systems man and cybernetics, part c-applications and reviews*, 41(5), pp. 674–681, 2011, doi: 10.1109/TSMCC.2010.2093127.

[4] SVÍTEK M. Wave probabilistic functions for quantum cybernetics, 2012, *IEEE transactions on systems man and cybernetics, part c-applications and reviews.* 42(2), pp. 233–240, 2012, doi: 10.1109/TSMCC.2011.2115237.

[5] VEDRAL V. Introduction to Quantum Information Science, *Oxford University Press*, 2006.

[6] KAUFFMAN S. Autonomous Agents, in John D. Barrow, P.C.W. Davies, and C.L. Harper Jr., eds., Science and Ultimate Reality: Quantum Theory, Cosmology, and Complexity, *Cambridge University Press.* 2004. ISBN 9780521831130.

[7] FEYNMAN R., LEIGHTON R., SANDS M. Feynman lectures of physics, *Addison Wesley Longman, Inc.*. USA, 1966.

[8] VOTRUBA Z., NOVÁK M. Alliance Approach to the Modelling of Interfaces in Complex Heterogenous Objects. *Neural Network World,* 2010, 20(5), pp 609–619.

[9] SVÍTEK M. Physics-Information Analogies, *Neural Network World*, 2018, 28(6), pp. 535–550, doi: 10.14311/NNW.2018.28.030.

[10] MOOS P., SVÍTEK M., NOVÁK M., VOTRUBA Z. Information Model of Resonance Phenomena in Brain Neural Networks. *Neural Network World*, 2018, 28(3), pp. 225–239, ISSN 1210-0552.

[11] SVÍTEK M.: Wave probabilistic information power, In: Neural Network World 2011, 21(3), pp. 269–276, ISSN 1210-0552.

[12] MALINOVSKÝ, V.: System Macromodel of Agricultural Building with Aim to Energy Consumption Minimization, AGRIS on-line Papers in Economics and Informatics, 10(1), 2018, pp. 25–35. ISSN 1804-1930. doi: 10.7160/aol.2018.100103.

[13] BOUCHNER P., HAJNY M., NOVOTNY S., et al. Car simulation and virtual environments for investigation of driver behavior, Neural Netw. World, 15(2), pp. 149–163, 2005.