



HYBRID MATRIX FACTORIZATION FOR RECOMMENDER SYSTEMS IN SOCIAL NETWORKS

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Abstract: Recommender systems have been well studied and applied both in the academia and industry recently. However, traditional recommender systems assume that all the users and items are independent and identically distributed. This assumption ignores the correlation of explicit attributes of both users and items. Aiming at modeling recommender systems more realistically and interpretably, we propose a novel and efficient hybrid matrix factorization method which combines implicit and explicit attributes, and can be used to solve the problem of cold start and recommender interpretation. Based on the MovieLens datasets, the experimental analysis shows our method is promising and efficient.

Key words: *recommender system, matrix factorization, hybrid factors, recommended interpretation*

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1. Introduction

As an important service provided through the Internet, the social network has become an important tool for users to participate in social activities and get information. A large number of data information, such as demographic characteristics, clicking, friends linking, login information and attributes of items concerned by users, exist in social networks and can be used for recommender systems, by which social network sites can provide personalized service, improve the adhesion from users, and users can improve the efficiency of getting personalized preference information. At present, personalized recommender systems have become an important application integrated by online social networks.

The precision of rating prediction is one of evaluation indexes in personalized recommendation systems. Studies have shown that the precision is related to both recommendation data and methods. In general, recommendation data sets are very sparse because of users' reluctance to offer their preference and the restriction of

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privacy protection [28]. Therefore, recommender methods have been recognized as the main means to improve the precision in academia. Matrix Factorization (MF) methods are widely used ones in rating prediction. And several matrix factorization methods [4, 11, 14, 15] have been proposed for recommender systems. All these methods focus on fitting the user-item rating matrix with low-rank approximations, and use it to make further rating predictions. The premise of which has a low-dimensional factor model that is only influenced by a small number of preference factors [11]. Basic MF (BMF) [24] methods only consider users' rating data, and assume that users and items latent factors are independent and identically distributed. This assumption ignores social interactions or connections among users and relevance among users and items explicit attributes. Considering the relationship among users and items, Ma H. etc. proposed the MF method with explicit and implicit social relation [16], which uses the relations as constraint of matrix factorization and improves the rating prediction precision of recommender systems. However, the method only shows that the relations among users and items has a positive influence on its overall prediction precision, and cannot indict the relationship between the special users and items. On the other hand, the method can not give any explanation of the recommendation and solve the cold start problem due to using latent factors.

In view of the above problem, A Hybrid Matrix Factorization (HMF) method is proposed in this article. In HMF, users or items factors matrix are composed of explicit and implicit attributes rather than latent factors. The main innovations in this paper are as follows: The correlation among users and items are retained in HMF, more information are utilized rather than the simple assumption independent and identically distributed of users and items factors; The explicit attributes of the user and the items are included in factors matrix, so HMF can be used to recommend new users and new items; A mapping from rating matrix to weights of explicit attributes is realized, and to some extent interpretation of recommender can be given.

The remainder of this article is organized as follows. In Section 2, we provide an overview of several major MF methods and related works. Section 3 presents HMF method which combines explicit and implicit attributes, and gives the solution. In Section 4, the results of an empirical analysis are presented, followed by the conclusions with future work in Section 5.

2. Related works

Recommender systems are software tools and techniques providing suggestions for items to be of use to a user [3, 17, 21], and emerged as an independent research area in the mid-1990s. Driven by Netflix prize especially, the interest about recommender systems has dramatically increased in recent years [12, 13]. According to different evaluation criteria, recommender systems can be divided into two types: ranking and rating [1, 6, 10]. The former returns a sorting list of items, which users might be interested in, and the latter predicts possible ratings of users to items. Ranking is more practical than rating, but the rating is easily achieved to evaluate the performance of different methods, and it can also be used to give the ranking list, for which the rating methods are widely used in evaluation about recommender

systems. Collaborative filtering methods are most classical and frequent ones used for rating prediction [2, 7, 10, 23]. The methods assume that similar users have the same preferences and similar items will be selected by same users, and use correlation or similarity among users and items to predict ratings or recommend items for users. Using the ratings instead of other information, such as demographic, items attributes, collaborative filtering methods can be widely applied in recommender systems for their protecting users' privacy well.

K Nearest Neighbour (KNN) and MF are general methods in collaborative filtering. KNN includes three main methods: user-based [2, 7, 10], item-based [23] and hybrid methods [26]. In KNN, the similarity values play a double role in neighborhood-based recommendation methods, allow the selection of trusted neighbors whose ratings are used in the prediction and provide the means to give more or less importance to these neighbors in the prediction as well. As the recommender data are so sparse that many methods of KNN focus on their similarity definition. Generally, users' preference would change dynamically with the time, while attributes of items remain unchangeable. Items-based methods are superior to the user-based ones in prediction precision [19]. MF methods are model-based ones. The basic MF method decomposes user rating matrix $\mathbf{R} \in \mathbb{R}^{m \times n}$ into users' latent factors matrix $\mathbf{U} \in \mathbb{R}^{m \times k}$ and items latent factors matrix $\mathbf{V} \in \mathbb{R}^{n \times k}$. A prediction rating \hat{r}_{ij} of a user i to a item j is $\hat{r}_{ij} = \mathbf{U}_i \cdot \mathbf{V}_j^T$. In the prediction stage, MF methods are highly efficient and have higher prediction accuracy than other rating methods which are not model-based. Therefore, MF methods are classical ones and widely used in prediction ratings.

Among MF methods, Probability Matrix Factorization (PMF) has been widely used [22, 24]. Because of sparse rating data, a matrix factorization algorithm with the regularized RSVD [11] was proposed by Netflix contest winners to prevent over-fitting data. Regularization can improve the precision of predicted ratings, and it is widely used in recommender systems. Considering users and items rating biases, Rendle etc. [20] use Biases Probability Matrix Factorization (BPMF) to eliminate the influence of biases. Because the purchase and rating may affect mutually, users may accept recommendation from others, and different items attributes factors may be correlated, Ma etc. [15, 16] proposed MF methods including users and items correlation, they use users' social relations and correlation among items attributes as constraints in matrix factorization stage. The methods improve the overall recommendation accuracy and coverage of recommendation items. But for a single specific user, the method cannot draw a conclusion as to whether the result is improved. Like general matrix decomposition, the methods can not give interpretation of results as using latent factors. In order to improve the interpretation of recommended results, Hernando etc. [8] proposed non negative matrix factorization method (NMF), in which, the value of latent factors is non-negative, and it avoids meaningless negative value. For improving efficiency, Ortega etc. [18] proposed group matrix factorization (GMF). In GMF, both users' preference and items categories are restricted to the limited class. In fact, GMF method belongs to the regularization method. In order to solve problem of data sparse, Jiang etc. [9, 29] use transfer learning methods, which integrate data from different fields to process the recommendation problem. The methods can better deal with the problem of data sparseness and improving the accuracy of recommendation. In

the above MF methods, as preference of users and items are represented by latent factors, new users and items can not be recommended and can not give an interpretation about results of recommender. HMF, which is proposed in this article, includes explicit and implicit attributes in factors matrix. Explicit attributes can be used not only to explain recommendation results, but also to solve the cold start problem.

3. Hybrid implicit and explicit attributes matrix factorization

3.1 Formulation of HMF

For users-items rating matrix $\mathbf{R} \in \mathbb{R}^{m \times n}$, a r_{ij} is i row and column j element in matrix \mathbf{R} , it represents rating of a user i to an item j . As rating data is very sparse, there is a large number of missing data in matrix. The object of probability MF methods is to decompose rating matrix \mathbf{R} into users latent factors matrix $\mathbf{U} \in \mathbb{R}^{m \times k}$ and items latent factors matrix $\mathbf{V} \in \mathbb{R}^{n \times k}$, and to make \mathbf{UV}^T and \mathbf{R} as close as possible, in which k is the number of latent factors, and $k \ll m, n$. In PMF methods, as \mathbf{U} and \mathbf{V} are represented by latent factors. They cannot give an interpretation about a result of recommender. Meanwhile, they assume that the attribute factors are independent and identically distributed, as a result, the effect of the correlation among users and items are not considered. HMF is different from others MF methods which use correlation of users and items explicit attributes as constrain. It directly represents correlation using explicit attributes factors which include in users or items factors matrix.

In HMF, explicit and implicit attributes factors are included in matrix \mathbf{U} , \mathbf{V} . Explicit attributes factors can reflect correlation among users and items, and implicit attributes factors can represent unknown or complex features which are difficult to indicate. When using explicit attributes, we might give an interpretation about recommender users or items. Here, users attributes matrix can be represented as $\mathbf{U} = [\mathbf{U}^{\text{ex}}, \mathbf{U}^{\text{im}}]$, in which $\mathbf{U}^{\text{ex}} \in \mathbb{R}^{m \times k_1}$ is an explicit attribute matrix block, and $\mathbf{U}^{\text{im}} \in \mathbb{R}^{m \times k_2}$ is an implicit attribute matrix block, k_1 is the number of explicit attributes, k_2 is the number of implicit attributes, $k = k_1 + k_2$ is the total number of factors in user matrix. Like users attributes matrix, items attributes matrix is represented as $\mathbf{V} = [\mathbf{V}^{\text{ex}}, \mathbf{V}^{\text{im}}]$.

As it is difficult to establish relations between users explicit attribute and items explicit attributes, we firstly consider a situation in which items contain the explicit attributes, and the user matrix is still using the latent factors matrix. Then, $\mathbf{V}_j = [\mathbf{V}_j^{\text{ex}}, \mathbf{V}_j^{\text{im}}]$, and the cost-function of the HMF method is defined as follows:

$$l(\mathbf{U}, \mathbf{V}) = \sum_{\substack{r_{ij} \in \mathbf{R} \\ r_{ij} \neq \emptyset}} (r_{ij} - \mathbf{U}_i \mathbf{V}_j^T).$$

The object function of HMF is as follows:

$$\min \sum_{\substack{r_{ij} \in \mathbf{R} \\ r_{ij} \neq \emptyset}} (r_{ij} - \mathbf{U}_i \mathbf{V}_j^T).$$

To avoid over fitting, we regularize \mathbf{U}_i and \mathbf{V}_j , Therefore, the objective function can be formulated as follows:

In which, λ is coefficient of regularization. As non-negative value of elements in items matrix is more explicable than arbitrary value. Then, the value of elements in \mathbf{U}_i , \mathbf{V}_j are constrained by non-negative, see Eq. (1);

$$\min_{s.t. v > 0} \sum_{\substack{r_{ij} \in \mathbf{R} \\ r_{ij} \neq \emptyset}} (r_{ij} - \mathbf{U}_i \mathbf{V}_j^T) + \lambda(\|\mathbf{U}_i\|^2 + \|\mathbf{V}_j\|^2). \quad (1)$$

In which, v is the value of elements in items factors.

3.2 Solution of HMF

Both stochastic gradient descent (SGD) [27] and alternating least squares (ALS) [25] are commonly used methods of matrix decomposition. In this paper, we use an improved stochastic gradient descent algorithm for HMF. First, we calculate the partial differences of cost-function respectively about \mathbf{U}_i and \mathbf{V}_j , see Eqs. (2), (3);

$$\frac{\partial l}{\partial \mathbf{U}_i} = -2\mathbf{V}_j(r_{ij} - \mathbf{U}_i \mathbf{V}_j^T) + 2\lambda \mathbf{U}_i, \quad (2)$$

$$\frac{\partial l}{\partial \mathbf{V}_j} = -2\mathbf{U}_i(r_{ij} - \mathbf{U}_i \mathbf{V}_j^T) + 2\lambda \mathbf{V}_j. \quad (3)$$

In HMF, \mathbf{V}_j includes two components \mathbf{V}_j^{im} and \mathbf{V}_j^{ex} . When using alternating least squares, \mathbf{U}_i , \mathbf{V}_j^{im} and \mathbf{V}_j^{ex} are updated iteratively as follows:

$$\mathbf{U}_i \leftarrow \mathbf{U}_i + \alpha(\mathbf{V}_j(r_{ij} - \mathbf{U}_i \mathbf{V}_j^T) - \lambda \mathbf{U}_i),$$

$$\mathbf{V}_j^{\text{im}} \leftarrow \mathbf{V}_j^{\text{im}} + \alpha(\mathbf{U}_i(r_{ij} - \mathbf{U}_i \mathbf{V}_j^T) - \lambda \mathbf{V}_j),$$

$$\mathbf{V}_j^{\text{ex}} \leftarrow \mathbf{V}_j^{\text{ex}}, \quad (4)$$

where α is a learning rate.

To ensure the factors values meaningful, when $v < 0$, we set $v = 0$, where $v \in \mathbf{V}_j^{\text{im}}$. The value of \mathbf{V}_j^{ex} always remain the same and cannot be changed. see Eq. (4). The value of users factors \mathbf{U}_i , corresponding explicit attributes of \mathbf{U}_j^{ex} , reflects the weights of users' i preference to special explicit attributes of items j .

3.3 Interpretation and cold start

In MF, as latent factors is used, recommender interpretation is impossible. On the other hand, as explicit attributes are not directly used in factors matrix of users and items, we cannot recommend new users or items according its attributes. It is known as the problem of cold start in recommender systems. But in HMF, explicit attributes are used, then interpretation can be given and new users or items can be recommended.

Users latent factor matrix \mathbf{U} can be divided into two parts, \mathbf{U}^{ep} and \mathbf{U}^{ip} . \mathbf{U}^{ep} corresponding to explicit attributes of items, \mathbf{U}^{ip} corresponding to an explicit attributes of items. The recommender weight of explicit attributes of items is calculated as follows:

$$w_{r_{ij}} = \frac{\mathbf{U}_i^{\text{ep}}(\mathbf{V}_j^{\text{ex}})^T}{\mathbf{U}_i^{\text{ep}}(\mathbf{V}_j^{\text{ex}})^T + \mathbf{U}_i^{\text{ip}}(\mathbf{V}_j^{\text{im}})^T}.$$

The weights represent extent of users preference to items. When the weights are greater than a certain threshold, we can recommend new items for user.

In order to solve problems of cold start, we can use explicit attributes of items to represent its matrix factors. Solving Eq. (1) with the ALS algorithm, we can get users preference to the items according its explicit attributes. And users preference matrix is updated as follows:

$$\mathbf{U}_i \leftarrow \mathbf{U}_i + \alpha(\mathbf{V}_j^{\text{ex}}(r_{ij} - \mathbf{U}_i(\mathbf{V}_j^{\text{ex}})^T) - \lambda \mathbf{U}_i).$$

As items facotr matrix are expressed with explicit attributes. Recommender interpretation is possible about items. Similarly, when the users matrix are represented with hybrid attributes factors, HMF can be used to recommend items for new users.

4. Experiments

4.1 Experiments data

In experiments, we select 100k dataset of MovieLens [5]. The data sets were collected by the GroupLens Research Project at the University of Minnesota. It consists of 100,000 ratings (1–5) from 943 users on 1682 movies. Each user has rated at least 20 movies; Simple demographic info for the users (age, gender, occupation, zip) and simple genres, title, URL info for items are included. This data has been cleaned up. Users who had less than 20 ratings or did not have complete demographic information were removed from this data set. About explicit attributes, we only select genres of movies. As genres include 19 fields, and number of movies with some special genres is few. We sort genres according its number of movies and get a list of genres.

4.2 Experiments evaluating indicator

There are many types of evaluation in recommender systems. Precision is commonly used to evaluate the performance of the recommendation method. Typi-

cally, the ratings \mathbf{R} are divided into a training set $TrainSet$ used to optimal factors matrix of user and items, and a test set $TestSet$ used to evaluate the prediction accuracy. Two popular measures of accuracy are the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are used, see Eqs. (5), (6);

$$MAE = \frac{\sum_{(u,i) \in TestSet} |r_{ui} - \hat{r}_{ij}|}{|TestSet|}, \quad (5)$$

$$RMSE = \sqrt{\frac{\sum_{(u,i) \in TestSet} (r_{ui} - \hat{r}_{ij})^2}{|TestSet|}}, \quad (6)$$

where $|TestSet|$ is the number of elements in testset.

4.3 Experiments results and analysis

In our Experiments, we test precisions of different number of explicit and implicit attributes, and 5-fold cross validation method is used. As BPMF method has higher accuracy than other MF ones, first, we use BPMF as a comparison method, and the parameters are set as: regularization $\lambda = 0.02$, learning rate $\alpha = 0.1$. The precision of BPMF with different number of latent factors are shown as Tab. I.

Num.	MAE	RMSE
5	0.7552	0.9779
10	0.7533	0.9621
15	0.7389	0.9437
20	0.7409	0.9483
25	0.7400	0.9467
30	0.7336	0.9436
40	0.7365	0.9400

Tab. I Precision of different number of latent factors using BPMF.

As seen form the Tab. I, when number of latent factors is smaller than 30, the precision will improve along with the increase of number of latent factors. MAE = 0.7336 is optimal results. When number of latent factors is 40, it can cause over-fitting phenomena and precision reducing.

Second, to confirm the HMF effectiveness, we set total number of factors as 5, and results of including different number of explicit factors are given. When number of explicit attributes is equal to 5, the items factor matrix only include explicit attributes. The parameters are set as: regularization $\lambda = 0.02$, learning rate $\alpha = 0.1$. When the number of explicit attributes is n in HMF, top- n attributes are selected from the genres sort list. The precision with different number of latent factors are shown as Tab. II.

As shown in Tab. II, when the number of total factors is 5, the results of HMF is better than BPMF which use latent factors. This is because HMF takes advantage of the correlation of items and constrains over-fitting caused by sparse rating data. As users' preferences are subtle, it is difficult to express it by only using explicit

Num.	MAE	RMSE
1	0.7351	0.9392
2	0.7316	0.9421
3	0.7396	0.9468
4	0.7404	0.9458
5	0.7532	0.9579

Tab. II Precision of different number of explicit factors using HMF.

attributes, and it is necessary to include implicit attributes in factors matrix. With ratio of number of explicit attributes increased in factors matrix, the correlation is greatly enhanced, and functions of implicit attributes are weakened, the precision will reduce as shown in Tab. II.

Last, we set the number of explicit attributes as 5, and total factors is from 5 to 30. The precision of fixed number of explicit factors using HMF are shown as Tab. III.

Num.	MAE	RMSE
5	0.7532	0.9579
10	0.7452	0.9523
15	0.7443	0.9529
20	0.7439	0.9531
25	0.7374	0.9431
30	0.7372	0.9411

Tab. III Precision of fixed number of explicit factors using HMF.

Comparing Tab. III with Tab. I, in the same number of factors, we can see that HMF method is better than BPMF method which only use latent factors methods. With ratio of number of explicit attributes reduced, precision improving is limited. It is because the weights of explicit correlation are reduced.

4.4 Recommender new items

When explicit attributes of new items are known, we can use item-based or HMF to recommend new items for user. In HMF, rating matrix is factorized to item matrix which includes explicit attributes and user matrix which can be regarded as the weight of items explicit attributes corresponding. Then, we can use users' preference matrix of HMF and new items explicit attributes to predict users ratings to items and recommend new items. In our experiments, items genre is used as explicit attributes of its factor matrix. We compare the performance of HMF and item-based methods, and the results is shown in Tab. IV.

From Tab. IV, we can see that HMF is superior to item-based method. It is because HMF can distinguish different weights of attributes, but item-based method set same values for all weights of item attributes. As HMF includes explicit attributes, it also can give an interpretation of recommender results. the item's

Method	MAE	RMSE
HMF	0.7532	0.8051
Item-based	0.9579	1.0501

Tab. IV Precision of HMF and Item-based for recommender new items.

explicit attributes which most affected the item to be recommended to the user can be illustrated.

5. Conclusion

In this paper, we proposed a novel and efficient probabilistic matrix factorization method which links ratings with explicit attributes of users or items. The method not only considers the correlation of explicit attributes, but also applicable to solve the problem of cold start and recommender interpretation. Experimental analysis on the MovieLens datasets shows the promising future of our proposed method. As the exponential growth of online social network sites continues, lots of data can be collected and used. Hybrid more types data will be an interesting research to improve quantity of recommender.

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References

- [1] ADOMAVICIUS G., TUZHILIN A. Toward the next generation of recommender systems : A survey of the state-of-the-art and possible extensions. *IEEE Transactions on Knowledge & Data Engineering*. 2005, 17(6), pp. 734–749, doi: [10.1109/TKDE.2005.99](https://doi.org/10.1109/TKDE.2005.99).
- [2] BREESE J.S., HECKERMAN D., KADIE C. Empirical analysis of predictive algorithms for collaborative filtering. In: *Proceedings of the Fourteenth Conference on Uncertainty in Artificial Intelligence*, Madison, Wisconsin. 2013, pp. 43–52.
- [3] CHEN W., ZHANG L.J., CHEN C., BU J.J. A Hybrid Phonic Web News Recommender System for Pervasive Access. In: *Proceedings of the International Conference on Communications and Mobile Computing*, Yunnan, China. IEEE, 2009, pp. 122–126, doi: [10.1109/CMC.2009.20](https://doi.org/10.1109/CMC.2009.20).
- [4] DEVARAJAN K., WANG G., EBRAHIMI N. A unified statistical approach to non-negative matrix factorization and probabilistic latent semantic indexing. *Machine Learning*. 2015, 99(1), pp. 137–163, doi: [10.1007/s10994-014-5470-z](https://doi.org/10.1007/s10994-014-5470-z).
- [5] HARPER F.M., KONSTAN J.A. The MovieLens Datasets: History and Context. *Transactions on Interactive Intelligent Systems*. 2015, 5(4), pp. 19–19, doi: [10.1145/2827872](https://doi.org/10.1145/2827872).

- [6] HERLOCKER J.L., KONSTAN J.A., TERVEEN L.G., RIEDL J.T. Evaluating collaborative filtering recommender systems. *Acm Transactions on Information Systems*. 2004, 22(1), pp. 5–53, doi: [10.1145/963770.963772](https://doi.org/10.1145/963770.963772).
- [7] HERLOCKER J.L., KONSTAN J.A., BORCHERS A., Riedl J. An algorithmic framework for performing collaborative filtering. In: *Proceedings of the International ACM SIGIR Conference on Research and Development in Information Retrieval*, Berkeley, Ca, USA. 1999, pp. 230–237.
- [8] HERNANDO A., BOBADILLA J., ORTEGA F. A non negative matrix factorization for collaborative filtering recommender systems based on a Bayesian probabilistic model. *Knowledge-Based Systems*. 2016, 97, pp. 188–202, doi: [10.1016/j.knosys.2015.12.018](https://doi.org/10.1016/j.knosys.2015.12.018).
- [9] JIANG M., CUI P., CHEN X., WANG F. Social Recommendation with Cross-Domain Transferable Knowledge. *IEEE Transactions on Knowledge & Data Engineering*. 2015, 27(11), pp. 3084–3097, doi: [10.1109/j.ins.2016.01.083](https://doi.org/10.1109/j.ins.2016.01.083).
- [10] KONSTAN J.A. Introduction to recommender systems: Algorithms and Evaluation. *Acm Transactions on Information Systems*. 2004, 22(1), pp. 1–4, doi: [10.1145/963770.963771](https://doi.org/10.1145/963770.963771).
- [11] KOREN Y., BELL R., VOLINSKY C. Matrix Factorization Techniques for Recommender Systems. *Computer*. 2009, 42(8), pp. 30–37, doi: [10.1109/MC.2009.263](https://doi.org/10.1109/MC.2009.263).
- [12] KOREN Y. Collaborative filtering with temporal dynamics. In: *Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining*, Paris, France. ACM, 2009, pp. 89–97, doi: [10.1145/1557019.1557072](https://doi.org/10.1145/1557019.1557072)
- [13] KOREN Y. Factor in the neighbors: Scalable and accurate collaborative filtering. *Acm Transactions on Knowledge Discovery from Data*. 2010, 4(1), pp. 2010, doi: [10.1145/1644873.1644874](https://doi.org/10.1145/1644873.1644874).
- [14] LEE D.D., SEUNG H.S. Learning the parts of objects by non-negativ matrix factorization. *Nature*. 1999, 401(6755), pp. 78–91, doi: [10.1038/44565](https://doi.org/10.1038/44565).
- [15] MA H., YANG H., LYU M.R., KING I. SoRec: Social Recommendation Using Probabilistic Matrix Factorization. In: *Proceedings of the ACM Conference on Information and Knowledge Management*, Napa Valley, California, USA. ACM, 2008, pp. 931–940.
- [16] MA H., KING I., LYU M.R. Learning to recommend with explicit and implicit social relations. *Acm Transactions on Intelligent Systems & Technology*. 2011, 2(3), pp. 135–136, doi: [10.1145/1961189.1961201](https://doi.org/10.1145/1961189.1961201).
- [17] MAHMOOD T., RICCT F. Improving recommender systems with adaptive conversational strategies. In: *Proceedings of the 20th ACM conference on Hypertext and hypermedia*, Torino, Italy. ACM, 2009, pp. 73–82, doi: [10.1145/1557914.1557930](https://doi.org/10.1145/1557914.1557930).
- [18] ORTEGA F., HERNANDO A., BOBADILLA J., KANG J.H. Recommending items to group of users using Matrix Factorization based Collaborative Filtering. *Information Sciences*. 2016, 345(C), pp. 313–324, doi: [10.1016/j.ins.2016.01.083](https://doi.org/10.1016/j.ins.2016.01.083).
- [19] PAPAGELOS M., PLEXOUSAKIS D. Qualitative analysis of user-based and item-based prediction algorithms for recommendation agents. *Cooperative Information Agents VIII*. Springer Berlin Heidelberg. 2004, pp. 781–789, doi: [10.1007/978-3-540-30104-2_12](https://doi.org/10.1007/978-3-540-30104-2_12).
- [20] RENDLE S., SCHMIDT-THIEME L. Online-updating regularized kernel matrix factorization models for large-scale recommender systems. In: *Proceedings of the 2008 ACM conference on Recommender systems*, Lausanne, Switzerland. ACM, 2008, pp. 251–258, doi: [10.1145/1454008.1454047](https://doi.org/10.1145/1454008.1454047).
- [21] RESNICK P., VARIAN H.R. Recommender systems. *Communications of the ACM*. 1997, 40(3), pp. 56–58, doi: [10.1145/245108.245121](https://doi.org/10.1145/245108.245121).
- [22] SALAKHUTDINOV R., MNIIH A. Probabilistic Matrix Factorization. *Advances in Neural Information Processing Systems*. 2015, pp. 1257–1264, doi: [10.1145/1390156.1390267](https://doi.org/10.1145/1390156.1390267).
- [23] SARWAR B., KARYPIS G., KONSTAN J., John R. Item-based collaborative filtering recommendation algorithms. In: *Proceedings of the 10th international conference on World Wide Web*, Hong Kong. 2011, ACM, pp. 285–295, doi: [10.1145/371920.372071](https://doi.org/10.1145/371920.372071).

- [24] TAKAMATSU S., SATO I., NAKAGAWA H. Probabilistic Matrix Factorization Leveraging Contexts for Unsupervised Relation Extraction. In: *Proceedings of the Advances in Knowledge Discovery and Data Mining 2011*, Shenzhen, China. 2011, pp. 87–99, doi: [10.1007/978-3-642-20841-6_8](https://doi.org/10.1007/978-3-642-20841-6_8).
- [25] TAKANE Y., YOUNG F.W., LEEUW J.D. Nonmetric individual differences multidimensional scaling: An alternating least squares method with optimal scaling features. *Psychometrika*. 1977, 42(1), pp. 7–67, doi: [10.1007/BF02293745](https://doi.org/10.1007/BF02293745).
- [26] WANG J., VRIES D., REINDERS A.P., MARCEL J.T. Unifying User-based and Item-based Collaborative Filtering Approaches by Similarity Fusion. In: *Proceedings of the 29th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval, SIGIR '06*, Seattle, Washington, USA. ACM, 2006, pp. 501–508, doi: [10.1145/1148170.1148257](https://doi.org/10.1145/1148170.1148257).
- [27] ZHANG T. Solving Large Scale Linear Prediction Problems Using Stochastic Gradient Descent Algorithms. In: *Proceedings of the Twenty-first International Conference on Machine Learning (ICML 2004)*. Banff, Alberta, Canada. ACM, 2004, pp. 919–926, doi: [10.1145/1015330.1015332](https://doi.org/10.1145/1015330.1015332).
- [28] ZHANG Z.Y., GUPTA B.B. Social Media Trustworthiness and Security: Overview and New Direction. *Future Generation Computer Systems*. 2016, pp. 1–12, doi: [10.1016/j.future.2016.10.007](https://doi.org/10.1016/j.future.2016.10.007).
- [29] ZHAO L., PAN S.J., XIANG E., ZHONG E., LU Z., YANG Q. Active transfer learning for cross-system recommendation. In: *Proceedings of the Twenty-Seventh AAAI Conference on Artificial Intelligence (AAAI'13)*, Bellevue, Washington. AAAI Press, 2013, pp. 1205–1211.