PARADOXES IN NUMERICAL CALCULATIONS

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Abstract: When solving problems of mathematical physics using numerical methods we always encounter three basic types of errors: modeling error, discretization error, and round-off errors. In this survey, we present several pathological examples which may appear during numerical calculations. We will mostly concentrate on the influence of round-off errors.

Key words: round-off errors, numerical instability, recurrence formulae, Gram-Schmidt orthogonalization

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Dedicated to Prof. Ivo Babuška on his 90th birthday

1. Introduction

The general computational scheme for solving problems of mathematical physics is sketched in Fig. 1. In general, three basic types of errors $e_0$, $e_1$, and $e_2$ are introduced.

Fig. 1 General computational scheme: The modeling error $e_0$ is the difference between physical reality and its mathematical model. The difference between the mathematical model and the discrete model is called the discretization error $e_1$. Finally, round-off errors are included in $e_2$. 

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