ISOLATED RUPTURE DEGREE OF TREES AND GEAR GRAPHS

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Abstract: The isolated rupture degree for a connected graph G is defined as $ir(G) = \max\{i(G-S) - |S| - m(G-S) : S \in C(G)\}$, where $i(G-S)$ and $m(G-S)$, respectively, denote the number of components which are isolated vertices and the order of a largest component in $G - S$. $C(G)$ denotes the set of all cut-sets of G. The isolated rupture degree is a new graph parameter which can be used to measure the vulnerability of networks. In this paper, we firstly give a recursive algorithm for computing the isolated rupture degree of trees, and determine the maximum and minimum isolated rupture degree of trees with given order and maximum degree. Then, the exact value of isolated rupture degree of gear graphs are given. In the final, we determine the rupture degree of the Cartesian product of two special graphs and a special permutation graph.

Key words: *Isolated rupture degree, vulnerability, ir-set, recursive algorithm, Cartesian product, gear graph*

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1. Introduction

Throughout this paper, a graph $G = (V, E)$ always means a simple connected graph with vertex set V and edge set E. The number of vertices $|V|$ is known as the order of G. For $v \in V$, we denote the degree of v by $d_G(v)$. The maximum degree of a graph G is denoted by $\Delta(G)$. A vertex set $S \subseteq V(G)$ is a cut set of G, if either $G - S$ is disconnected or $G - S$ has only one vertex. $C(G)$ denotes the set of all cut-sets of G. For $S \subseteq V(G)$, let $\omega(G - S)$, $i(G - S)$ and $m(G - S)$, respectively, denote the number of components, the number of components which are isolated vertices and the order of a largest component in $G - S$. We shall use $\lfloor x \rfloor$ for the largest integer less than or equal to a real number x. A Δ-*edge* is an edge which joins two vertices of degree Δ. A *leaf* is a vertex of degree 1. An edge incident with a leaf is called a *leaf-edge*. An edge is said to be *subdivided* when it is replaced by a path of length two connecting its ends, and the internal vertex in this path is a new vertex. A subset S of V is called an *independent set* of G if no two vertices of S are adjacent in G. An independent set S is called a maximum independent set

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