A CRITIQUE OF THE STANDARD COSMOLOGICAL MODEL

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tutorial

Abstract: According to the standard cosmological model, 27 % of the Universe consists of some mysterious dark matter, 68 % consists of even more mysterious dark energy, whereas only less than 5 % corresponds to baryonic matter composed from known elementary particles. The main purpose of this paper is to show that the proposed ratio 27 : 5 between the amount of dark matter and baryonic matter is considerably overestimated. Dark matter and partly also dark energy might result from inordinate extrapolations, since reality is identified with its mathematical model. Especially, we should not apply results that were verified on the scale of the Solar System during several hundreds of years to the whole Universe and extremely long time intervals without any bound of the modeling error.

Key words: *Dark matter, dark energy, antigravity, modeling error, manifold, extrapolation, cosmological parameters*

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Never identify any model with reality.

1. Introduction

In 1584, Giordano Bruno wrote the treatise *De l'Infinito, Universo e Mondi,* where he introduced the hypothesis that the universe is infinite and that each star looks like our Sun. This statement is often considered as the origin of modern cosmology. Isaac Newton and many others envisioned the Universe as the Euclidean space $\mathbb{E}^n$ for dimension $n = 3$.

In 1900, Karl Schwarzschild [60, p. 66] was probably the first person to realize that the Universe could be non-Euclidean, and moreover to have a finite volume. He assumed that it is described by a large three-dimensional manifold, the *hypersphere*

$$\mathbb{S}^3_r = \{(x, y, z, w) \in \mathbb{E}^4 \,|\, x^2 + y^2 + z^2 + w^2 = r^2\}. \tag{1}$$

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For simplicity, we shall omit the subscript r if $r = 1$. Recall that an *n-dimensional manifold* is a set of points such that each point has an open neighborhood that can be continuously mapped to an open set in $\mathbb{E}^n$ with continuous inverse. The hypersphere (1) has at any point and any direction the same curvature $1/r$, i.e., the inverse of the radius of the osculation circle. Similarly $\mathbb{E}^3$ has at any point and any direction zero curvature. This enables us to represent the Universe as having a high homogeneity and isotropy on large scales. The discovery and development of non-Euclidean geometries are discussed in the survey paper [12].

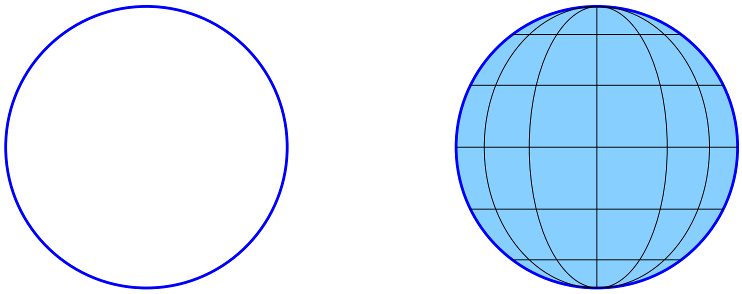

Fig. 1 *The unit circle on the left is the sphere* $\mathbb{S}^1 = \{(x, y) \in \mathbb{E}^2 \mid x^2 + y^2 = 1\}$. *The surface of the unit ball on the right is the sphere* $\mathbb{S}^2 = \{(x, y, z) \in \mathbb{E}^3 \mid x^2 + y^2 + z^2 = 1\}$.

In [60] on page 67, Schwarzschild even considered the Universe as having a hyperbolic geometry which is for $r > 0$ usually modeled by the hypersurface (cf. Fig. 2 and [56, p. 826])

$$\tilde{\mathbb{H}}^3_r = \{(x, y, z, w) \in \mathbb{E}^4 \mid x^2 + y^2 + z^2 - w^2 = -r^2\} \tag{2}$$

with the Minkowski metric. Let us emphasize that w in formula (2) is not time as it could seem from the often used and confusing notation $t = w$ (see e.g. [69, p. 95]). Namely, if w would be time, then the associated space manifold for constant values of w would only have dimension two. Hence, it could not model our Universe.

Recall that the Gaussian curvature of a smooth two-dimensional surface in $\mathbb{E}^3$ is defined as the product of curvatures in the two main perpendicular directions. For instance, the sphere $\mathbb{S}^2_r$ has a positive Gaussian curvature $r^{-2} = r^{-1} \cdot r^{-1}$, since all its osculation great circles have radius r.

Already in 1901, David Hilbert proved (see [24]) that there does not exist a smooth surface in $\mathbb{E}^3$ (bounded or unbounded) without boundary and with a constant negative Gaussian curvature. A survey of two-dimensional surfaces in $\mathbb{E}^3$ with a constant negative Gaussian curvature is given in [41]. However, all these surfaces have a singularity, like an edge or a cusp point, and thus they are not globally smooth. For instance, the surface that arises by rotation of the tractrix curve in $\mathbb{E}^3$ has the Gaussian curvature (-1) everywhere. It looks like a trumpet with a circular edge (see [32]) and cannot be smoothly extended beyond this edge so that the Gaussian curvature would remain (-1).

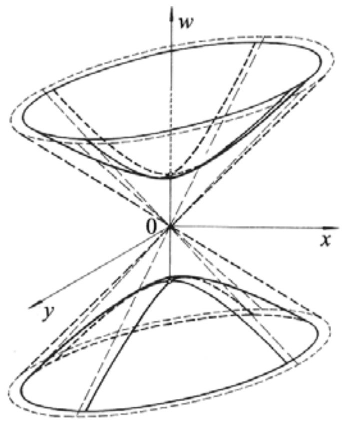

Fig. 2 *Two-sided hyperboloid* $x^2 + y^2 - w^2 = -1$.

Recall that the sectional curvature at a given point is a function of two linearly independent vectors v and w, and it expresses the Gaussian curvature of the two-dimensional submanifold given by the pair of tangential vectors v and w (see [28, p. 143]). If the sectional curvature is constant for all such pairs, then we say that the manifold has a *constant space curvature*. For any $n \geq 2$ there exist just three kinds of the maximally symmetric manifolds $\mathbb{S}_r^n$, $\mathbb{E}^n$, and $\mathbb{H}_r^n$ with $r > 0$ that have the constant space curvature $1/r^2, 0$, and $-1/r^2$, respectively. The exact definition of maximally symmetric manifolds (including $\mathbb{H}_r^n$) is based on the so-called Killing vectors which would require the introduction of many technical details [71].

It is not easy to visualize the geometry of $\mathbb{H}^3$. Let us emphasize that (2) is only a mathematical model of the maximally symmetric hyperbolic manifold $\mathbb{H}_r^3$. In [12], another five mathematical models of $\mathbb{H}^3$ are presented.

Recall that an *isometry* is a continuous mapping $f : M \to M$ whose inverse exists and is continuous and preserves distances on a manifold M, i.e., $\rho(f(A), f(B)) = \rho(A, B)$ for all $A, B \in M$, where ρ is a metric on M. Hilbert in [24] in fact proved that there is no isometric embedding of the hyperbolic plane $\mathbb{H}^2$ into the three-dimensional space $\mathbb{E}^3$, while the sphere $\mathbb{S}^2$ is isometrically imbedded into $\mathbb{E}^3$ (see Fig. 1). The manifolds $\mathbb{H}^2$ and $\mathbb{H}^3$ can be isometrically embedded into $\mathbb{E}^6$ and $\mathbb{E}^{12}$, respectively, but it is not known whether these dimensions can be reduced, see [8] and [11]. The manifold $\mathbb{H}^3$ is thus a rather exotic mathematical object.

2. The standard mathematical cosmological model

According to the *Einstein cosmological principle*, the Universe on large scales is homogeneous and isotropic at a fixed time. The homogeneity is expressed by a translation symmetry (i.e., the space has at any point the same density of mass, pressure, temperature, etc.). The isotropy is expressed by a rotational symmetry (i.e., there are no preferred directions and the observer is not able to distinguish one direction from another by local physical measurements). Hence, the Universe for a fixed time is modeled by the maximally symmetric manifolds $\mathbb{S}^3_r$, $\mathbb{E}^3$, or $\mathbb{H}^3_r$.

In 1922, Alexander Friedmann [21] derived from Einstein's equations for a perfectly symmetric space, which is homogeneous and isotropic for each fixed time instant, a nonlinear differential equation of the first order for an unknown *expansion function* $a = a(t) > 0$,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}, \tag{3}$$

where $\rho = \rho(t) > 0$ denotes the mean mass density of the Universe at time t, $G = 6.674 \cdot 10^{-11}$ m^{3}kg^{-1}s^{-2} is the gravitational constant, Λ is the cosmological constant, $c = 299\,792\,458$ m/s is the speed of light in the vacuum, k/a^2 is the space curvature, and k is the curvature index (normalized curvature). The value $k = 1$ corresponds to the hypersphere $\mathbb{S}^3_r$ with variable radius (see (1))

$$r = r(t) = a(t).$$

The case $k = 0$, which was not considered by Friedmann in [21], corresponds to $\mathbb{E}^3$.

In this way Friedmann described the dynamical behavior of the Universe as an alternative to Einstein's stationary Universe [18]. In 1924, he published another paper [22], where the negative curvature index $k = -1$ is considered. However, equation (3) was derived only for a negative density of mass (see [22, p. 2006]) and it is not clear how to satisfy such a paradoxical assumption. Fortunately, we may examine equation (3) also for $k = -1$ and $\rho \geq 0$. If $k = -1$ then the space at a fixed time can be modeled by the hyperbolic hypersurface (2), which in older literature is sometimes understood as a *pseudosphere* with imaginary "radius" ir for $r = a(t) > 0$ (cf. Fig. 2). In the standard cosmological model the curvature index may attain only three values

$$k \in \{-1, 0, 1\}. \tag{4}$$

Friedmann was the first who found that the Universe could have "zero radius" in the past (see [21, footnote 11]). Later Georges E. Lemaître developed the Big Bang theory in [39]. His theory is at present in agreement with the cosmological redshift of galaxies and their obvious evolution at cosmological distances, with the character of microwave background radiation, and the existence of primordial light elements that arose during the first several minutes after the Big Bang [72]. The half-life period of a solitary neutron, which is not imprisoned in an atomic nucleus, is only 611 seconds. This fact supports the origin of primordial helium and lithium.

Recall that cosmological distances in the observable Universe are usually expressed by the *redshift*

$$z = \frac{\lambda - \lambda_0}{\lambda_0},$$

where λ_0 is the wavelength of a particular spectral line under normal conditions when the light source and the observer are in quiet, whereas λ is the wavelength of the corresponding light from the observed celestial object (quasar, galaxy, cluster of galaxies, etc.). If $z < 0$, we talk about the *blueshift*.

In 1917, Albert Einstein included a positive cosmological constant Λ to his equations of general relativity to avoid gravitational collapse and to save his model of the stationary Universe [18]. However, the resulting solution of equation (3) is not stable, i.e., any small deviation from constant $a = a(t)$ will cause either a gravitational collapse, or expansion (see [46, p. 746]). Although the theory of general relativity was invented to explain various paradoxes of the Newtonian theory of gravitation for large velocities, masses, densities, etc., the Friedmann equation (3) for $\Lambda = 0$ can easily be formally derived from the Newtonian theory (cf. [45]).

In 1929, Edwin Hubble found in [26] that the Universe on large scales expands and that the recession speed of galaxies v from our Galaxy is approximately proportional to their distance d, that is

$$v \approx H_0 d, \tag{5}$$

where H_0 is the *Hubble constant.* By investigating 22 galaxies, Hubble [26] found that $H_0 = 550$ km/(s Mpc), where 1 pc $= 3.086 \cdot 10^{16}$ m. Nevertheless, the Hubble constant was not first introduced by Hubble as it is often claimed. Already in 1927, Lemaître calculated its value as 625 km/(s Mpc) in [39, p. 56]. He derived it from the Strömberg list [66, p. 200] of cosmological red and blueshifts of extragalactic nebulae after subtraction of the speed of the Solar System with respect to the Milky Way. Let us still point out that already in 1915, Vesto M. Slipher [63] discovered a dominance of redshifts over blueshifts of extragalactic nebulae in our neighborhood. At that time he did not know that they are galaxies.

The present value of the Hubble constant is

$$H_0 \approx 70 \text{ km}\,\text{s}^{-1}\text{Mpc}^{-1} \approx 2.33 \cdot 10^{-18} \text{ s}^{-1}. \tag{6}$$

The Planck Collaboration report [52, p. 30] presents several of its possible values, for instance,

$$H_0 = 67.3 \pm 1.2 \text{ km}\,\text{s}^{-1}\text{Mpc}^{-1} \quad \text{and} \quad H_0 = 73.8 \pm 2.4 \text{ km}\,\text{s}^{-1}\text{Mpc}^{-1}, \tag{7}$$

that are probably influenced by large systematic errors. Since the expansion speed of the Universe was larger in the past (see Fig. 3), we define the *Hubble parameter*

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{8}$$

so that $H(t_0) = H_0$, where t_0 is the age of the Universe. The function $a = a(t)$ is sometimes also called the *scaling parameter.* By (8) the expansion function $a = a(t) > 0$ is increasing at present as $H_0 > 0$. The function $a = a(t)$ appears in the Friedmann–Lemaître–Robertson–Walker metric which defines the corresponding spacetime manifold (see [49], [56], [70]).

It is not easy to establish the current value of the Hubble parameter $H(t)$, since we always observe only the past. In our close neighborhood, the measurement of $H_0 = H(t_0)$ is impaired by local movements of galaxies. On the other hand, it is very difficult to extrapolate reliably the current value of H_0 from long-distance objects (e.g. from microwave background radiation which traveled to us for over 13 Gyr, see [51, p. 16]).

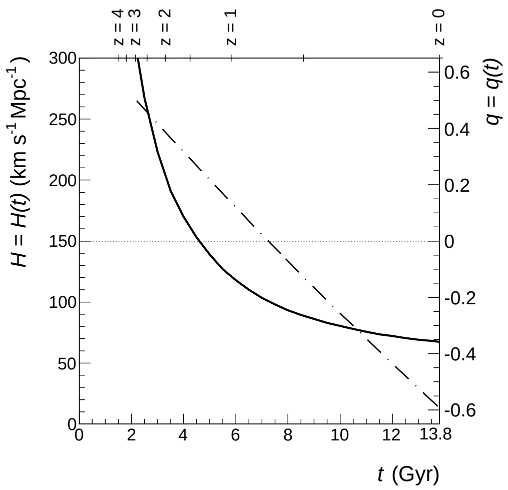

Fig. 3 *The behavior of the Hubble parameter $H = H(t)$ is sketched by the solid line according to the model from [50]. The dashed-dotted line stands for the corresponding deceleration parameter $q = -1 - \dot{H}/H^2$ that was derived by means of numerical differentiation. The lower horizontal axis shows time in* Gyr *since the Big Bang. In the upper horizontal axis we see the associated cosmological redshift z.*

3. Strange behavior of cosmological parameters

In literature in cosmology, division of equation (3) by the square $H^2 = (\dot{a}/a)^2 \geq 0$ is usually done without any preliminary warning that we may possibly divide by zero which may lead to various paradoxes. Then for all t we get the equality for three dimensionless parameters

$$1 = \Omega_{\rm M}(t) + \Omega_\Lambda(t) + \Omega_{\rm K}(t), \tag{9}$$

where

$$\Omega_{\rm M}(t) = \frac{8\pi G\rho(t)}{3H^2(t)} > 0, \quad \Omega_\Lambda(t) = \frac{\Lambda c^2}{3H^2(t)}, \quad \Omega_{\rm K}(t) = -\frac{kc^2}{\dot{a}^2(t)}, \tag{10}$$

and $\Omega_{\rm M}$ is the *parameter of density of dark and baryonic matter*, Ω_Λ is the *parameter of density of dark energy*, and $\Omega_{\rm K}$ is the *parameter of density of spatial curvature*, see [25, p. 71], [49]. The Planck Collaboration [52] calls $\Omega_{\rm K}$ the curvature parameter. The function $\rho_c(t) = 3H^2(t)/(8\pi G)$ is called the *critical density* for historical reasons, since if $\Lambda = 0$, then $k = 0 \Leftrightarrow \rho = \rho_c$, $k = 1 \Leftrightarrow \rho > \rho_c$, and $k = -1 \Leftrightarrow \rho < \rho_c$.

1) Let us first study the behavior of cosmological parameters in the case of the Einstein stationary universe, where $\dot{a}(t) = 0$ for all t (cf. Fig. 4). Then from (8) we have $H(t) = 0$. Even though nothing dramatic happens, by (10) the parameter

of the mass density $\Omega_{\mathrm{M}}(t) = \infty$ for all t. We should write more precisely that it is not well defined. Reasonably defined physical values should not attain infinite values.

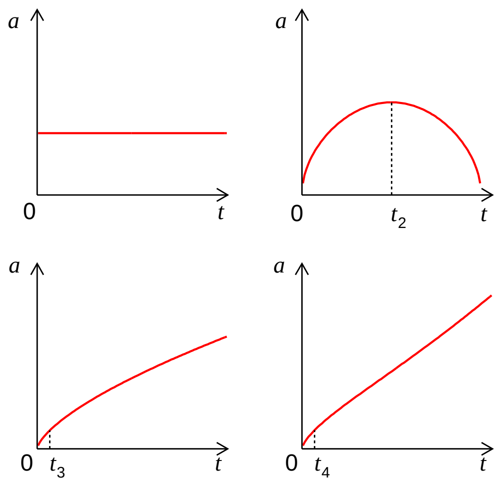

Fig. 4 *The expansion function for the stationary universe, the cyclic universe, the universe with zero cosmological constant, and for a currently accepted expansion of the universe with a positive cosmological constant.*

2) Consider now another classical model, the so-called cyclic or pulsating or oscillating universe. Assume for a moment that its expansion stops at some time $t_2 > 0$ and then starts to shrink (see Fig. 4). Then $\dot{a}(t_2) = 0$ and by (10) for $\Lambda > 0$ the parameter of the density of dark energy, which should accelerate the expansion of the Universe, is equal to $\Omega_\Lambda(t_2) = \infty$. However, the Universe starts to collapse. Even in a close neighborhood of the point t_2, where we do not divide by zero, the behavior of cosmological parameters is bizzare, since their values rapidly grow beyond all bounds.

3) In the model with zero cosmological constant and $k = -1$ it is assumed that the expansion function tends to infinity for $t \to \infty$ and is strictly concave for $t > t_3 > 0$ (see Fig. 4 and [46, p. 735]). Hence, the derivative $\dot{a}$ and also its square are decreasing functions. By (10) the parameter of spatial curvature $\Omega_{\mathrm{K}} > 0$ increases for $t \to \infty$, whereas the space curvature k/a^2 tends to zero. From points 1)–3) we observe that all three cosmological density parameters (10) do not have appropriate names.

4) A somewhat more curious behavior of the parameter Ω_{K} is obtained for the currently accepted expansion function. Similarly as in the previous point 3) we shall consider only $t > t_4 > 0$, where t_4 denotes the origin of the microwave background radiation. According to the measurements by the 2011 Nobel Prize Winners [53], the expansion function $a(t)$ is strictly concave over the interval circa $(t_4, 9)$ Gyr and then changes to a strictly convex function on the interval $(9, 14)$ Gyr. In other

words, the function $\dot{a}$ is first increasing and then decreasing (see Fig. 4). From this it follows by (10) that the parameter of density of spatial curvature $\Omega_{\rm K}(t)$ is not a monotonic function, even though the Universe expands continually. The absolute value of the parameter of the density of spatial curvature $|\Omega_{\rm K}| > 0$ on the interval $(t_4, 9)$ Gyr increases for $k \neq 0$, but the spatial curvature tends to zero with increasing time. We again see that the name for $\Omega_{\rm K}$ was not appropriately selected.

Let us further note that by the theory of inflation, the Universe expanded exponentially during a very short time instant after the Big Bang, i.e., the expansion function $a = a(t)$ was strictly convex. Then it was strictly concave and then surprisingly it was again strictly convex by point 4).

4. Excessive extrapolations

Without any exception every equation of mathematical physics has some limitations on the size of the investigated objects. For instance, the standard heat equation approximates very well the true temperature in solids of size about 1 meter, which can be verified by direct measurements (see [37]). However, in applying the heat equation on the atomic level in the cube with edge 10^{-10} m, we get nonsensical numbers, as well as in the cube with edge 10^{10} m, which would immediately collapse into a back hole (note that the diameter of our Sun is $1.4 \cdot 10^9$ m.) The same is true for linear elasticity equations, semiconductor equations, supraconductivity equations, Navier–Stokes equations for fluids, Maxwell's equations, Korteweg-de Vries equations, magneto-hydro-dynamic equations, and so on. Similarly, we cannot apply Keplerian laws on scales of 10^{-10} m or the Schrödinger equation on objects that have the size of a cat. Therefore, in any calculation we have to take care of the modeling error.

In spite of that, when deriving the Friedmann equation (3), the Einstein equations (containing among other the Newton gravitational constant G) are applied to the whole Universe. This is considered as a platitude and almost nobody deals with the question, whether it is justified to perform such a fearless extrapolation without any observational support, since general relativity was "checked" only for much smaller scales like the Solar System (slowdown of electromagnetic waves and bending of light in the gravitational field of the Sun, measuring the curvature of spacetime near the rotating Earth by means of the Lense-Thirring precession effect, perihelion advance of Mercury's orbit, etc., see [46], [61]). Note that galaxies have a diameter on the order of 10^{10} astronomical units and the Universe has at least five orders of magnitude more. Hence, the Friedmann equation (3) was derived under a considerably incorrect extrapolation. So it cannot describe reality well. This seems to be the main misconception of the current cosmology.

In applying the standard cosmological model various "delicate" limits are performed: $a \to 0$, $a \to \infty$, $t \to 0$, $t \to \infty$, ... (see e.g. [3], [46], [49], [71]). In this way, the amount of dark matter and dark energy is derived up to three significant digits and the age of the Universe is derived even up to four significant digits as 13.82 Gyr (see [52]). Note that the age of some small stars in our Galaxy is estimated to be at least 13.6 Gyr independently of cosmological models [10], i.e., these stars should have been formed about $t_1 = 220$ million years after the Big Bang, which is too short a time period. According to current models, the temperature of clouds of molecular hydrogen should be about 10 K, which is necessary for star formation by Jeans' criteria when gravity dominates over pressure. The temperature of the microwave background radiation (see Fig. 5) was much higher, $2.73(z+1) \approx 50$ K, where the cosmological redshift $z \approx 17.5$ corresponds by [50] to the time t_1.

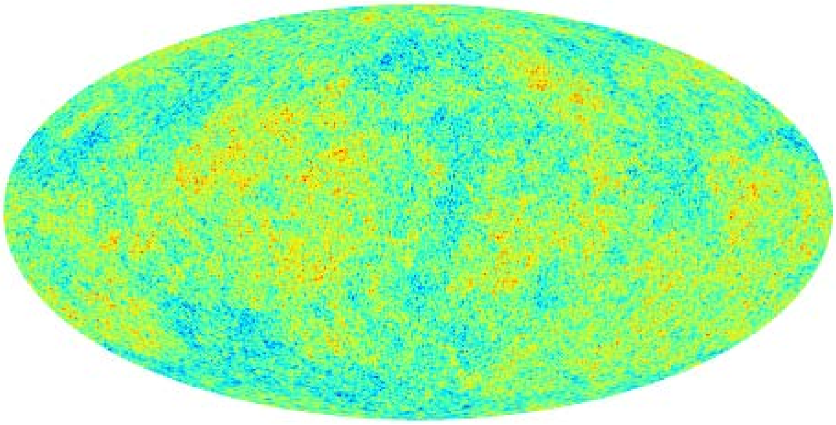

Fig. 5 *Tiny fluctuations in temperature* ≈ 2.73 K *of the cosmic microwave background corresponding to the cosmological redshift* $z = 1089$. *This radiation arose when the Universe was* 1090 *times smaller and had a mean temperature almost* 3000 K *(photo satellite Planck).*

For the time being, only the two coefficients $H_0 = H(t_0)$ (see (7)) and $q_0 = q(t_0) \approx -0.6$ (see [55, p. 110]) of the Taylor series of the expansion function were measured (with very low precision),

$$\begin{aligned} a(t) &= a(t_0) + \dot{a}(t_0)(t - t_0) + \frac{1}{2}\ddot{a}(t_0)(t - t_0)^2 + \dots \\ &= a(t_0)(1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots). \end{aligned} \tag{11}$$

where the *deceleration parameter* $q = -\ddot{a}a/(\dot{a})^2$ depends on the second derivatives of $a = a(t)$. (Originally, cosmologists believed that the expansion of the Universe slows down, and therefore, they did not introduce the acceleration parameter but a deceleration parameter.) However, the first three terms of the Taylor expansion at the point t_0, which corresponds to present time, cannot well describe the behavior of the expansion function in the far past (e.g. Fig. 6 for 10 Gyr ago).

We cannot reliably estimate the remainder of the Taylor series on the whole domain of definition, since the first derivatives of the expansion function $a = a(t)$ were extremely large just after the Big Bang. To see this, denote the estimated age of the Universe by $t_0 = 13.82$ Gyr. The microwave background radiation appeared $t_1 = 380\,000$ years after the Big Bang (see e.g. [19], [52]). Then for the measured

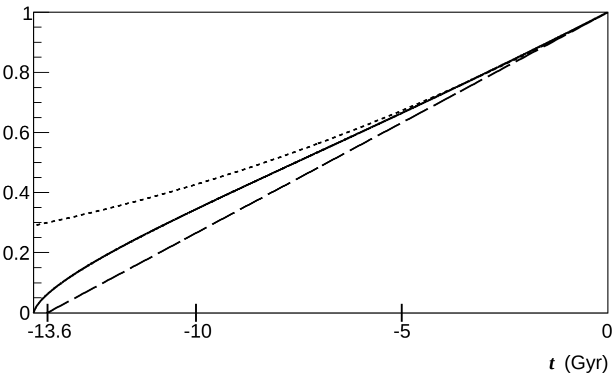

Fig. 6 *The assumed behavior of the normalized expansion function $a(t)/a(0)$. The time variable is shifted for simplicity so that $t_0 = 0$ corresponds to the present time. The values on the horizontal axis are given in* Gyr. *The quantities on the vertical axis are relative, with no physical dimensions. The lower dashed graph corresponds to the linear function $1 + H_0 t$ from (11) on the interval $[-1/H_0, 0]$, where $1/H_0 = 13.6$ Gyr is the Hubble time. The upper dotted graph shows the quadratic function $1 + H_0 t - \frac{1}{2} q_0 H_0^2 t^2$ with $q_0 = -0.6$. The middle solid graph illustrates the behavior of the normalized expansion function according to data from [50] for $\Omega_{\mathrm{M}} = 0.317$, $\Omega_\Lambda = 0.683$, and $H_0 = 67.15$ km/(s Mpc). We observe that the accelerated expansion differs little from the linear expansion during the last few* Gyr.

cosmological redshift $z = 1089$ we find that

$$\frac{t_1 a(t_0)}{t_0 a(t_1)} = \frac{t_1}{t_0}(z+1) = \frac{380\,000 \cdot 1090}{13.82 \cdot 10^9} = \frac{1}{33.3},$$

where the first equality follows directly from (see [46, p. 730], [49, p. 96]) the definition of redshift. From this we get

$$33.3 \cdot \frac{a(t_0)}{t_0} = \frac{a(t_1)}{t_1},$$

and thus the mean expansion speed of the Universe on the interval $(0, t_1)$ was 33.3 times larger than on the interval $(0, t_0)$. This indicates that the expansion function had a much larger derivative after the Big Bang than today and that it was strictly concave on some subinterval (cf. Figs 6 and 7).

Since the product $\rho(t)a^3(t)$ is constant during the time period when matter dominates over radiation, equation (3) takes the equivalent form

$$\dot{a}^2 = Aa^2 + B + \frac{C}{a} \tag{12}$$

with time independent constant coefficients $A = \Lambda c^2/3$, $B = -kc^2$, and $C > 0$. From such a simple ordinary differential equation far-reaching conclusions about the deep past and the far future are made in [3], [49], ... Since the initial condition $\dot{a}(t_0)/a(t_0) = H_0$ is known, we may solve equation (12) forward and also backward in time. For the time period, when radiation dominates over matter, the term D/a^2 is added to the right-hand side of equation (12).

Further, we have to emphasize that the Friedmann equation (3) was derived only for the gravitational interaction. However, shortly after the Big Bang, electromagnetic forces that are 40 orders of magnitude higher played an important role. Before that also even stronger nuclear forces surely had an influence on the initial values of the true expansion function. Although nongravitational forces are investigated on large accelerators, their behavior in an extremely strong gravitational field right after the Big Bang is not known. In other words, the Friedmann equation can hardly describe the evolution of our Universe for small $t > 0$.

At present it is believed that $a(t) \to \infty$ for $t \to \infty$. By (9) and (10) for $k \le 0$ it follows that $\frac{1}{3}\Lambda c^2 < H^2(t)$ for arbitrary time. From this and (8) we observe that also the time derivative of the expansion function grows beyond all bounds if Λ is a positive constant. By contradiction we find that $\dot{a}(t) \to \infty$ for $t \to \infty$ in an infinite universe (hyperbolic or Euclidean).

5. Nonuniqueness of the notion universe

The term "universe" is used in cosmology in various meanings: true spacetime, true space (i.e. spacetime for a fixed time), and the observable universe, which is seen as a projection on the celestial sphere. These are three different objects. Their mathematical models are also three completely different manifolds (see Fig. 7). Altogether, we are discussing six different objects, for which the terminology is not fixed yet. The first three contain real matter, whereas the other three are only their abstract mathematical idealizations.

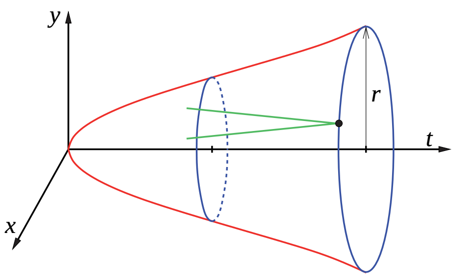

Fig. 7 *The model of spacetime for $k = 1$ is illustrated in red, blue stands for space (i.e. the model of the universe) for fixed time instants, and the model of the observable universe is in green. The space dimensions are reduced by two.*

In accordance with the Einstein cosmological principle from Section 2, we shall understand by the *universe* a cross-section of spacetime for a fixed time instant, i.e., the universe will be an isochrone in spacetime for constant t. For instance, if the curvature index is positive, the corresponding model of the universe is the hypersphere $\mathbb{S}_r^3$ for some fixed radius $r = r(t) > 0$, which is a three-dimensional manifold in the four-dimensional Euclidean space $\mathbb{E}^4$ (cf. Fig. 1). The associated model of spacetime in $\mathbb{E}^5$ has dimension four and the model of the observable universe has dimension three (cf. Fig. 7).

All six above-mentioned objects (which are often called only "universe") have to be carefully distinguished; otherwise we may come to various confusions. The observable universe is not homogeneous, since for different cosmological redshifts z it has a different mass density. Thus, it is an entirely different object than the universe as space. In the observable universe some cosmologists incorrectly try to measure angles α, β, γ in some large triangle to ascertain the spherical, Euclidean, or hyperbolic geometry of the universe by means of their sum $\alpha + \beta + \gamma$. Such a triangle has to be considered in the universe (space), in which we see only our close neighborhood (strictly speaking, only the one point at which we are situated). This limits our ability to perform such measurements.

6. Fritz Zwicky's postulation of dark matter

According to the method of baryonic acoustic oscillations and recent measurements of the Planck satellite (see [51], [52, p. 11]) the parameter of the mass density in the standard cosmological model is equal to

$$\Omega_{\mathrm{M}} = \Omega_{\mathrm{DM}} + \Omega_{\mathrm{BM}} \approx 0.32, \quad \Omega_{\mathrm{DM}} \approx 0.27, \quad \Omega_{\mathrm{BM}} \approx 0.05, \tag{13}$$

i.e., 27 % of the Universe consists of dark matter (DM) and 5 % consists of baryonic matter (BM), from which less than 1 % is made up of luminous matter. In the next three sections, we will show that these values do not fit with reality in the Coma galaxy cluster and our Galaxy. These are much smaller objects than the whole Universe.

In 1933, Fritz Zwicky [74] postulated the existence of dark energy after discovering large velocities of galaxies in the Coma cluster A1656 (see Fig. 8). With the help of classical Newtonian mechanics he derived a very simple relation for the so-called *virial mass* of the cluster,

$$M = \frac{5Rv^2}{3G}, \tag{14}$$

where R is its radius and v is the root-mean-square speed of all galaxies with respect to the center of mass of the cluster. According to actual data [1], [7], [15], [36],

$$R = 4.58 \cdot 10^{22} \text{ m}$$

and

$$v = 1686 \text{ km/s}. \tag{15}$$

Relation (14) thus yields almost ten times larger virial mass

$$M = 3.25 \cdot 10^{45} \text{ kg} \tag{16}$$

than the luminous mass

$$\mathcal{M} \approx 3.3 \cdot 10^{44} \text{ kg} \tag{17}$$

estimated from the Pogson equation, see [32] or [36] for details. Zwicky in [74] and [75] even obtained a more than two orders of magnitude larger value of M than $\mathcal{M}$. However, can we claim on the basis of such a trivial relation as (14) that dark matter really exists?

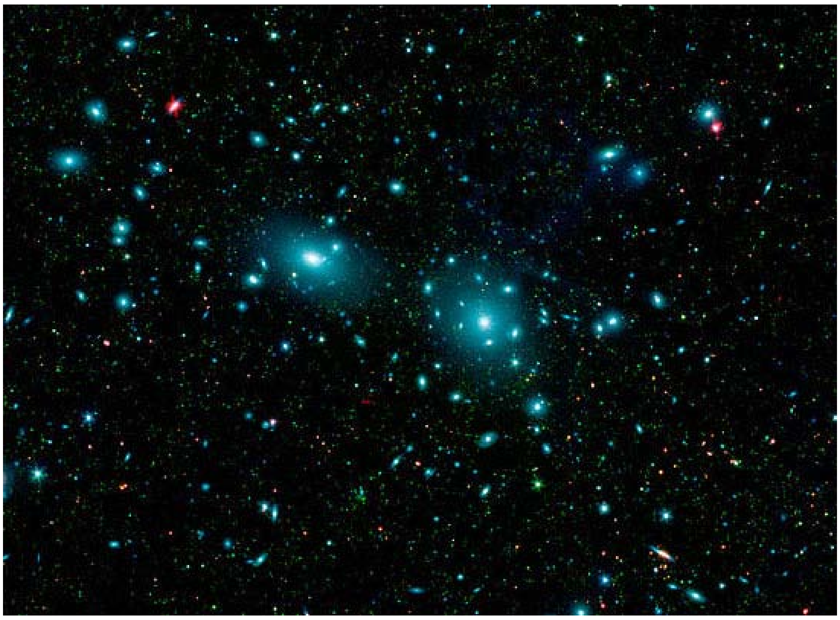

Fig. 8 *Large galaxy cluster Abell 1656 in the constellation Coma Berenices. In the middle there are two supergiant elliptic galaxies NGC 4889 and NGC 4874 which are 10 times more massive that the Milky Way (photo NASA).*

Zwicky became well aware that he needed to make many simplifications; otherwise he could not calculate anything. For instance, he assumed that galaxies are distributed uniformly, that the Virial Theorem holds exactly, and that gravitation has an infinite propagation speed. He substituted the spacetime curved by approximately one thousand galaxies by Euclidean space (see Fig. 9 and 10). He replaced galaxies of diameter of about 10^{10} au by mass points. Such approximations do not allow one to consider angular moments of rotating galaxies which surely contribute to the total angular momentum. Tidal forces among galaxies cannot be included as well.

Formula (14) was derived under many other simplifying assumptions. This list of assumptions is given in [36]. In postulating the existence of dark matter on the basis of such a trivial algebraic relation as (14), a large modeling error was surely made.

Let us briefly analyze some contributions to the modeling error. By considering a nonuniform galaxy distribution which better corresponds to reality than the uniform distribution, the coefficient $\frac{5}{3}$ from (14) can be reduced by 20–25 % (see [36] for a detailed proof).

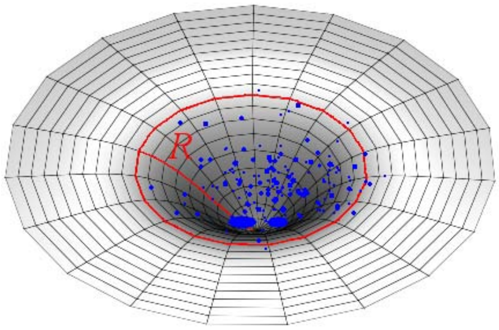

Fig. 9 *Deformation of spacetime due to the high density of galaxies in a cluster of radius R. The circumference of the circle with radius R is smaller than $2\pi R$.*

The measured redshift $z = 0.023$ of the Coma cluster can be assumed by (5) to be linearly proportional to the distance d, since $v \approx cz$ for small redshifts. In this way, the distance

$$d \approx 100 \text{ Mpc} \tag{18}$$

of the cluster from us was deduced from (6) by the formula

$$z = \frac{H_0}{c} d = 0.023. \tag{19}$$

The Coma cluster subtends the angle $\beta = 1.7^\circ$ in the celestial sphere. The corresponding radius

$$R = d \sin \tfrac{1}{2}\beta \tag{20}$$

can be then decreased, since the observation angle β is deformed due to the gravitational selflensing effect of the Coma cluster (see Fig. 10). By the famous formula for the bending angle (see e.g. [47] and [65]), we have

$$\phi = \frac{4GM}{c^2 R} \approx 2 \times 10^{-4} \text{ rad } \approx 0.7',$$

where $\phi = (\beta - \alpha)/2$ (see Fig. 4) and M is given by (16). This value represents about 1 % of 1° corresponding very roughly to the angular radius $\beta/2$ of the Coma cluster. Hence, R in (14) should be about 1 % smaller. However, the angle ϕ would be smaller when M in (16) is reduced.

The distance d as given in (18) is also overestimated due to the gravitational redshift of the Coma cluster, which has to be subtracted from the total measured redshift z. Each photon has to overcome not only the potential hole of its mother star, but also a much deeper potential hole of the corresponding galaxy, and the potential hole of the entire cluster, too (see Fig. 9). According to [13, p. 10], the gravitational redshift of the two large central galaxies of the Coma cluster (see Fig. 8) is about 61 km/s, which is about 1 % of the recession speed

$$\overline{v} \approx 6877 \text{ km/s}, \tag{21}$$

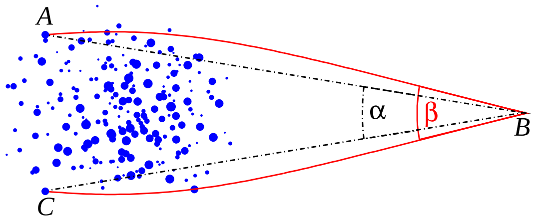

Fig. 10 *A schematic illustration of the selflensing effect. The observation angle* $\beta = \widetilde{\sphericalangle} ABC$ *is larger than the angle* $\alpha = \sphericalangle ABC$ *due to the bending of light caused by the gravitation of a galaxy cluster itself.*

where $\overline{v} = cz$. Although the redshifts of galaxies near the boundary of A1656 are about 20 km/s, this again leads to the overestimation of the distance d of A1656 from us and thus also of its radius (20), speed (15), and total mass (14).

The speed (21) is more than 2 % of the speed of light. In [36, p. 15] we showed that the distance d in (18) is also overestimated by 1 % also due to the relativistic effects of observed high velocities of galaxies.

The expansion speed of the Universe, which is characterized by the Hubble parameter $H = H(t)$, is decreasing with time (see Fig. 3). According to [50], its value at $z = 0.023$ is more than 1 % larger than the present value H_0. Hence, relation (19) should be replaced by

$$z = \frac{H(t)}{c} d = 0.023.$$

If $H(t)$ is larger than H_0, then d in (18) has to be smaller. Hence, also the radius R in (20) is again slightly overestimated by 1 %.

The root-mean-square speed v appearing in (14) can be decreased as well taking into account dark energy, gravitational aberration, finite speed of gravitational interaction, curved space time, etc. Since v in (14) is *squared*, the mass (16) can be reduced at least by a factor of two (see [36] for details). In summary, we obtain that the total mass of the cluster is at most five times larger than its luminous mass (17).

Recently Tutukov and Fedorova [67] found that the intergalactic medium of galaxy clusters contains 30–50 % of the total number of stars in the cluster. Moreover, by [2], [9], and [68] clusters of galaxies contain five times more baryonic matter in the form of hot gas producing X-rays than baryonic matter contained in galaxies. Consequently, the large velocities of galaxies in the Coma cluster observed by Zwicky have an entirely natural explanation by means only of baryonic matter. In the next section we will also show that the high amount of dark matter in (13) is exaggerated.

7. Vera Rubin's postulation of dark matter

Vera Rubin found that spiral galaxies have "flat" rotational curves (see [58]). On the basis of this fundamental discovery she developed in the 1970s her own theory of rotational curves of galaxies. She observed that stars orbit too fast about the galactic center and thus galaxies should have much more invisible matter than visible matter to hold the galaxy together.

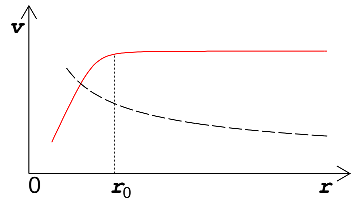

Fig. 11 *The dashed line illustrates a decrease of velocities of Keplerian orbits depending of the distance r from the center of a spiral galaxy. The solid line stands for an idealized rotational curve (which is flat for $r > r_0$) whose shape was discovered by Vera Rubin.*

Let us examine her hypothesis in more detail. Consider a point test particle with mass m (typically it will be a star) and let $M \gg m$ be the mass of another point that generates a gravitational field of central force. Assume that the test particle has speed v and a circular orbit about the center with radius r. Then from Newton's gravitational law and the formula for centripetal force, Rubin [36] easily derived that

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}, \quad \text{i.e.} \quad v = \sqrt{\frac{GM}{r}}. \tag{22}$$

Thus the speed v of a test particle of circular orbit is proportional to $r^{-1/2}$. Such orbits are called *Keplerian* (see Fig. 11). According to Vera Rubin [59, p. 491],

. . . , the stellar curve does not decrease as is expected for Keplerian orbits.

To clarify this paradox we have to realize that a spiral galaxy does not possess a gravitational field of central force except for a close neighborhood of the center, where e.g. in our Galaxy the stars S1, S2, . . . orbit the central black hole Sgr A* according to Keplerian laws. The mass of Sgr A* is 3.5 million times the Sun's mass which is less than 0.002 % of the total mass of the Galaxy (cf. (26)). On the other hand, in the Solar System, 99.85 % of its mass is concentrated in the Sun. Planets have almost no influence on their orbits, since their movements are controlled mainly by the central force of the Sun. On the contrary, trajectories of stars in a galactic disk are essentially influenced by neighboring stars, since the central bulge contains only about 10 % of all stars of the Galaxy. *Keplerian decrease of rotational curves is thus not justified.*

Denote by $M(r)$ the *baryonic mass* inside a ball of radius r and center in the middle of our Galaxy. Vera Rubin [58, p. 7] found in neighboring galaxies almost constant velocities of order $v \approx 200$ km/s for $r > r_0$, where r_0 is typically several kpc (see Fig. 11). On radii smaller than r_0, spiral galaxies rotate at a roughly constant angular speed (like an LP or DVD). The radius of the visible part of the Galaxy disk is about

$$R = 16 \text{ kpc} = 4.938 \cdot 10^{20} \text{ m} \tag{23}$$

and our Sun orbits about the galactic center at the speed

$$v_\odot = 230 \text{ km/s} \tag{24}$$

at a distance equal to about one-half of R. Stars orbiting at $r > r_0 \approx 3$ kpc should have approximately the same speed as $v_\odot$ due to the expected flat rotational curve (see Fig. 11).

At the end of the last century astronomers somewhat surprisingly believed that only 3 % of all stars are red dwarfs (see [6, p. 93]). However, at present we know that about 70 % of all stars are red dwarfs of the spectral class M. Vera Rubin evidently could not know about the existence of such a huge amount of red dwarfs. For this dramatic increase, we are indebted to the much better sensitivity of space telescopes. In this way, the estimated baryonic matter in our Galaxy highly increased. Denoting the Sun's mass by

$$M_\odot = 2 \cdot 10^{30} \text{ kg},$$

we find by [43, pp. 393–394] that the baryonic mass of all 400 billion stars in the Milky Way is

$$175 \cdot 10^9 M_\odot = 3.5 \cdot 10^{41} \text{ kg}, \tag{25}$$

taking into account all stars of luminosity classes I–V, i.e., supergiants, giants, subgiants, and all stars from the main sequence of the Hertzsprung–Russell diagram. The value (25) is based on the Hipparcos data from our near neighborhood of about several hundreds of parsecs from us, see Tab. I. The Harvard spectral classification [64] contains similar data that will be improved by the Gaia mission for more distant stars (cca 8 kpc) from the centre of the Milky Way up to the edge on the opposite side. Note that the accuracy essentially depends on the observed magnitude and extinction.

Spectral class	O	B	A	F	G	K	M	white dwarfs
Mass in $M_\odot$	25	5	1.7	1.2	0.9	0.5	0.25	0.7
Number in billions	10^{-5}	0.3	3	12	26	52	**270**	35
Product	≈ 0	1.5	5.1	14.4	23.4	26	**67.5**	24.5

Tab. I *Distribution of stars in the Milky Way according to spectral class. The second row shows the corresponding mass of a typical star in Solar mass units $M_\odot$. The number of stars from the associated class divided by 10^9 is given in the third row. The last row contains the baryonic mass of the whole class in billions of Solar masses.*

For the time being we cannot reliably establish what is the contribution to $M(R)$ from infrared dwarfs of new spectral classes: L – red-brown, T – brown, Y – black (the notation is not yet uniquely established), and exoplanets, whose luminosity is small. For instance, in 2013 Kevin Luhman discovered a binary brown dwarf whose distance from the Sun is only 6.5 ly. Another brown dwarf WISE J085510.83-071442.5 is 7.2 ly from us.

Large massive stars (giants and supergiants) live only a short time. After their death there remain many compact objects such as black holes, neutron and quark stars — MACHO (Massive Compact Halo Objects). They increase the value of $M(r)$ as well. In the galactic disk and bulge there exists also a large amount of nonluminous baryonic matter in the form of gas and dust (see Fig. 12). According to [43, p. 353], the amount of interstellar matter (without hypothetical dark matter) is estimated to be 10 % of the total mass of stars. Therefore, by (25)

$$M(R) \geq 3.85 \cdot 10^{41} \text{ kg} = 1.93 \cdot 10^{11} M_{\odot}. \tag{26}$$

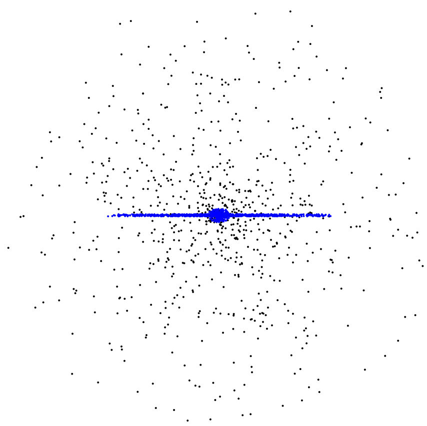

Fig. 12 *A lateral view of a spiral galaxy. The central bulge is surrounded by a flat disk and a sparse spherically symmetric halo that is filled by neutral hydrogen and helium, old stars, and globular clusters.*

Now let us concentrate the baryonic matter inside the ball of radius R into one central point, which is by [4, p. 149] and [32, p. 31] equivalent to a ball with spherically distributed mass. Then from relations (22), (23), and (26) we obtain that the circulating speed of stars

$$v = \sqrt{\frac{GM(R)}{R}} \geq \sqrt{\frac{6.674 \cdot 10^{-11} \cdot 3.85 \cdot 10^{41}}{4.938 \cdot 10^{20}}} = 228 \cdot 10^3 \text{ (m/s)} \tag{27}$$

on the Galaxy edge is really comparable with the measured speed (24). Although formula (27) is only approximate, to postulate 5–6 times more dark matter than baryonic matter as in (13) to keep the Galaxy gravitationally together seems to be highly overestimated. A detailed calculation is given in [32, p. 103].

Moreover, the following statement supporting large observed velocities holds:

Theorem. A particle orbiting a mass point along a circular trajectory with radius R has smaller speed than if it would orbit a flat disk with radius R and the same mass arbitrarily symmetrically distributed.

The proof is given in [32, p. 100]. Here we outline only the main idea. Consider two arbitrarily small volumes with masses $m_1 = m_2$ placed symmetrically with respect to the disk plane (see Fig. 13). Then the total force F acting on the test particle with mass m is smaller than the force $\overline{F}$, by which m acts on their projected arbitrarily small volumes. Denoting the distance between m_i and m by d, and its orthogonal projection by b, we get

$$F = G\frac{2m_1 m}{d^2} \cdot \frac{b}{d} \quad \text{and} \quad \overline{F} = G\frac{2m_1 m}{b^2}.$$

We see that the ratio of forces $\overline{F}$ and F is equal to the third power of the fraction d/b (cf. Fig. 13),

$$\overline{F} = \left(\frac{d}{b}\right)^3 F \geq F.$$

The corresponding speeds are proportional to these forces.

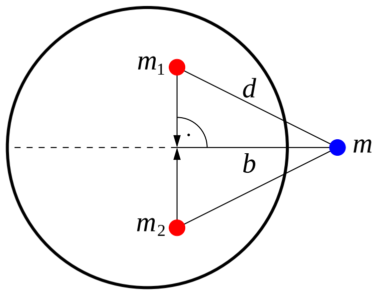

Fig. 13 *A ball with symmetrically distributed mass with respect to the horizontal plane acts on the test particle with mass m by a smaller force than the total mass of the ball projected orthogonally to the disk plane marked by the dashed line.*

8. Is dark matter merly a modeling error?

It is very probable that Newton's law of gravity on large cosmological scales approximates reality only very roughly. Therefore, we should not accept results of various Newtonian numerical simulations (like e.g. the Millennium simulation) which usually have thousands of lines of code and which seek to prove that without dark matter galaxies could not form after the Big Bang.

At present, several modifications of Newtonian mechanics have been developed, e.g. MOND (Modified Newtonian Dynamics) [40] and its relativistic generalization TeVeS (Tensor-Vector-Scalar) [5]. These theories try to explain consequences that are attributed to dark matter by means of another form of the gravitational law. On the other hand, there exist many papers (e.g. [20], [23], [27], [48], [62]) showing that on scales of galactic disks the Newtonian theory of gravitation is still a very good approximation of reality and it need not be modified. Moreover, the existence of dark matter need not be assumed. However, note that galaxies have negligible sizes compared to the observable Universe, where the Newtonian theory should not be applied.

Nowadays there is a large discussion concerning what dark matter is. The discrepancy of some model with reality does not mean that dark matter really exists, since the model can be wrong. Many sophisticated detectors (CDMS, DAMA/LIBRA, ADMX, ...) were constructed, but for the time being no dark matter has been detected. Also the large hadron collider in CERN did not find any new particles that could explain dark matter.

The influence of dark matter in the Solar System was not observed [48], even though the Sun is a large gravitational attractor. Thus it seems that dark matter, if it exists, is not able to dissipate its inner energy, and therefore cannot be concentrated at the Sun's neighborhood. The observed oscillations of stars perpendicularly to the galactic plane can be explained by classical Newtonian mechanics without dark matter (see [48]).

On the other hand, Douglas Clowe in his paper [14]: *A direct empirical proof of the existence of dark matter* shows an example of the collision of two galaxy clusters, where the intergalactic gas is disturbed, whereas galaxies continue in an unchanged direction together with dark matter which is "detected" by gravitational lensing. The regions with dark matter are artificially colored on the basis of some numerical simulations. However, we are not able to measure tangential components of the velocities of these clusters to prove that the collision really happened. Due to the large density of galaxies, the effect of dynamical friction should be observed among galaxies, but it is not. Both the clusters from [14] have almost the same size and they lie in one line together with clouds of dark matter which is from a statistical view point very improbable. In general, they should have different sizes and their trajectories should not lie in one line.

9. Dark energy versus cosmological constant

Generally, the prevailing conviction is that dark energy is some mysterious substance which is responsible for the accelerated expansion of the Universe. In the present cosmology, the Lambda Cold Dark Matter (ΛCDM) model is mainly preferred. According to data measured by the Planck satellite [52] the present parameter of the density of dark and baryonic matter is almost 32 % and the parameter of the density of dark energy about 68 %, more precisely,

$$\Omega_{\mathrm{M}} \approx 0.3175, \quad \Omega_{\Lambda} \approx 0.6825, \quad \Omega_{\mathrm{K}} \approx 0. \tag{28}$$

However, it is not said how to define this percentage for $\Lambda < 0$ or $\Omega_{\mathrm{K}} < 0$.

To derive the values (28), the method of baryonic acoustic oscillations [19] in the fluctuations of the microwave background radiation was applied. The image of the Milky Way was carefully removed from Fig. 5. Note that the microwave background radiation was continuously deformed by means of weak gravitational lensing of many galaxies and their clusters for more than 13 billion years. On the basis of such noisy data an extrapolation from $z = 1089$ to the present is made by means of equation (3). In this way relations (28) and also (7) were obtained.

From the previous three sections we know that the amount of dark matter (if it exists) is essentially not six times larger than the mass of baryonic matter as suggested in (13). Therefore, the value $\Omega_\Lambda \approx 0.6825$ in time $t_0 \approx 13.82$ Gyr is probably also far from reality. More precisely, we should say that the estimated age of the Universe derived from the ΛCDM model for the parameters (28) is $t_0 \approx 13.82$ Gyr. The true age can be completely different.

From the relations (28) we observe that the sum of the measured values $\Omega_{\mathrm{M}}(t_0)$ and $\Omega_\Lambda(t_0)$ is approximately equal to 1. This does not allow us to claim that from (9) and (10) it follows that $k = 0$ and that the true space is flat (i.e. infinite Euclidean) as it is often stated at present. Even though the sum would be

$$\Omega_{\mathrm{M}}(t_0) + \Omega_\Lambda(t_0) = 1.000000000000000001,$$

we still have a bounded universe that can be described by the sphere (1) with an incredibly large radius. Such a space is locally almost Euclidean, but finite. There is a big difference between a bounded and unbounded space. Moreover, the sphere $\mathbb{S}^3_r$ has an entirely *different topology* than proclaimed flat space $\mathbb{E}^3$.

The manifolds $\mathbb{E}^3$ and $\mathbb{H}^3_r$ have infinite volume. The Universe could not be finite at the beginning and then jump to infinite space. Moreover, it is difficult to imagine that the true infinite universe at every instant after the Big Bang would have at all its points the same temperature, pressure, density, etc., even though the theory of inflation is involved (see the argument just above Section 4). These quantities should have arbitrarily large values after the Big Bang as required by the cosmological principle. In this way, information would have an infinite speed of its propagation. Therefore, the most probable model of our Universe seems to be the hypersphere $\mathbb{S}^3_r$.

The physical dimension of the cosmological constant Λ is m^{-2}, since the left-hand side of (3) has dimension s^{-2}. Cosmologists describe it as the density of energy which has another physical dimension in the SI units (International System of Units), namely

$$\mathrm{kg\,m^{-1}s^{-2}}.$$

From relation (10) it is obvious that in quantities defining the parameter of density of dark energy $\Omega_\Lambda(t)$, the kilogram (kg) does not appear. Can we thus talk about density of energy?

We can easily verify that the physical dimension of the fraction $\frac{c^4}{G}\cdot\mathrm{m}^{-2}$ is the same as the density of energy in the units $\mathrm{kg\,m^{-1}s^{-2}}$. In the system $c = 1$ and $G = 1$ this is the same physical dimension as Λ has, since we may arbitrarily exchange kilograms, seconds, and meters using some appropriate multiplicative constants. In such a system, force, velocity, and power are dimensionless and we may evaluate energy and also time in kilograms. It is true that many relations will be much simpler in these restricted units, but the constants c and G in equation (3) are not equal to unity. Therefore, Λ cannot be interpreted as density of energy in the system SI.

Another possibility is to consider only the case $c = 1$. In this system we may define the density of energy by the relation $\rho_\Lambda = \Lambda/(8\pi G)$, since meters and seconds may be arbitrarily exchanged. This is again not the density of energy in the system SI.

Why should a single constant Λ truly model the accelerated expansion of the real Universe. Isn't it an oversimplification or too crude approximation? The standard cosmological model assumes that the expansion of the Universe is manifested only globally and not locally. However, in [31], [32], [33], and [38], we give several factual examples showing that the Solar System and also galaxies expand very slowly by a speed comparable with the Hubble constant H_0. This, of course, contradicts the law of energy conservation. Also other papers [16], [17], [42], and [73] derive that dark energy acts locally. In [30] and [34] we claim that gravitational aberration, which has a repulsive character, contributes to dark energy and thus has an influence on the expansion of the Universe. However, the local expansion cannot be described by a single constant, since it depends on position and time. Its average values are not described by a fundamental constant. Therefore, we should rather consider a time dependent function $\Lambda = \Lambda(t)$ (like the Hubble parameter $H(t)$ which also depends on time).

10. Conclusions

Current cosmological models are often identified with reality. On the basis of the simple standard cosmological model it is categorically claimed that our Universe is flat and that it consists of 68 % of dark energy, 27 % dark matter and 5 % baryonic matter. We showed that dark matter and partly also dark energy can be explained as a modelling error of the Friedmann model.

The Newtonian theory of gravitation is formulated so that the law of conservation energy holds. However, the real Universe is designed so that the total amount of energy slowly but continuously increases, since its expansion is accelerating. All small deviations from the Newton theory are successively accumulated. For instance, an extremely small deviation $\varepsilon > 0$ during one year may cause after one billion years a quite large and detectable value of $10^9\varepsilon$ which is then interpreted as dark energy.

In cosmology, we also often meet the following argumentation. Distances between galaxies increase and thus the entire Universe was concentrated at one point in the past (see e.g. [49], [72]). This implication is wrong from a mathematical point of view. As a counterexample it enough to take the everywhere increasing function

$$a(t) = C_1 + C_2\mathrm{e}^{C_3 t}, \quad t \in (-\infty, \infty) \quad \text{(with positive constants } C_1, C_2, C_3\text{)},$$

which is not zero — nor arbitrarily close to 0 as t approaches $\pm\infty$.

No two different points in the Universe which is marked by blue in Fig. 7 are causally connected. On the other hand, the observable universe, which is marked by green in Fig. 7, is causally connected with our present time and position represented by the cone vertex. The present speed of expansion of the Universe at time t_0 should thus depend on the density of mass in the past, since gravitation has a finite speed of propagation. For instance, baryonic matter, which slows the expansion, should be influenced by mass density at all previous time periods. Hence, the expansion function should be described by an equation whose solution depends on history. i.e., on all values $a(t)$ for $t \in (0, t_0)$. However, the Friedmann equation (3) does not have this property. It does not contain any delay given by the finite speed of gravitational interaction. It is only an ordinary differential equation whose solution on the interval (t_0, ∞) depends only on the value of the expansion function at point t_0 and not on the history. This is another drawback of the standard cosmological ΛCDM model.

Alexander Friedmann applied the Einstein equations to the whole Universe. Of course, when in 1922 he published his famous paper [21], he had no idea about the size of the Universe, since galaxies were discovered only in 1924 by E. Hubble. Thus, it seems that cosmologists nevertheless solve the normalized Friedmann equation (9) obtained by invalid extrapolations very exactly (up to four significant digits, see (28)).

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Curriculum Vitae

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Lawrence Somer was born in New York in 1948. During the period 1966–1970 he studied mathematics at Cornell University in Ithaca, New York, for his B.A. degree. He got the M.A. degree in mathematics in 1972 and the Ph.D. (advisor E. T. Parker) in 1985 from the University of Illinois. During the period 1986–2009 he was Professor of Mathematics at The Catholic University of America in Washington, D.C. After that he became Professor Emeritus in Mathematics there. He deals with number theory and is the coauthor of several monographs (Springer 2001, 2011, Academia 2009, 2011, Nova Sci. Publishes 2012). He is a member of the Mathematical Association of America, American Mathematical Society and of the editorial board of the journal *Fibonacci Quarterly.*

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