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# CONSTRUCTION OF THE POLYGONAL FUZZY NEURAL NETWORK AND ITS APPROXIMATION BASED ON $K$ -INTEGRAL NORM

Guijun Wang\*, Xiaoping Li†

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**Abstract:** The concept of an  $n$ -equidistant polygonal fuzzy number is introduced to avoid the complexity of the operations between fuzzy numbers. Firstly, the properties of linear operations and the convergence of  $n$ -equidistant polygonal fuzzy numbers are discussed, the method how to change a fuzzy number into an  $n$ -equidistant polygonal fuzzy number is shown. Next, for given a  $\hat{\mu}$ -integrable polygonal fuzzy valued function, an  $n$ -equidistant polygonal fuzzy valued function is constructed. By introducing the definition of  $K$ -quasi-additive integral and  $K$ -integral norm, the universal approximation of polygonal fuzzy neural network are studied. The final result indicates that the polygonal fuzzy neural network still possess universal approximation to an integrable system.

Key words: *Polygonal fuzzy numbers, quasi-additive integral, integrable polygonal fuzzy valued functions, integral norm, polygonal fuzzy neural network, universal approximation*

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## 1. Introduction

A fuzzy neural network (FNN) is an organic combination of an artificial neural network and fuzzy techniques, that form a hybrid intelligent system with both intelligent information processing and adaptability. In real life, it can effectively handle natural language messages, and there are more data messages of digital type than language messages. Thus, we can obtain data messages with corresponding input-output relationship of a fuzzy system by measurement data and transmission. Buckley [2–3] gave a conjecture when he studied the universal approximation of the regular FNN in 1994, and predicted the regular FNN is a universal approximation of the continuously increasing fuzzy function class. Since then, the network is

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\*Guijun Wang, Corresponding author, School of Mathematics Science, Tianjin Normal University, Tianjin, 300387, China, E-mail: [tjwgj@126.com](mailto:tjwgj@126.com)

†Xiaoping Li, School of Management, Tianjin Normal University, Tianjin, 300387, China, E-mail: [lxpmath@126.com](mailto:lxpmath@126.com)