QUANTUM SUBSYSTEMS CONNECTIONS

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Abstract: Paper presents the results in quantum informatics where two or more quantum subsystems are connected. For modelling the links amongst quantum subsystems the quantum quasi-spin is the most important parameter. We derive a quantum quasi-spin from the condition of logical requirement for the unambiguousness of wave probabilistic function assigned into quantum subsystem. With respect to these results we can define information bosons with integer quasi-spin, information fermions with half-integer quasi-spin and information quarks with third-integer quasi-spin. The methodology can be extended to other variants of quasi-spin.

1. Introduction

Models of complex systems are based on knowledge from information science, statistics or the knowledge that has been gathered over the years in classical physics, a specialized part of which is called information physics. Currently, a number of interesting results have been discovered in the field of quantum information science [4], taking as their basis the foundations of quantum physics [1] and using for modeling of complex systems those principles [2, 8] that do not arise in classical physics, such as entanglement and quantization [7, 10].

1.1 Wave probabilistic functions

Let us imagine that we are flipping a coin, so that every toss comes out as heads or tails. Someone else, who is assigned the role of an observer, is counting the frequency of the individual coin tosses and is estimating the probability of the phenomenon of it landing heads or tails in a simple manner, by counting the number of times it has fallen as heads or tails in the past, and by dividing that number by the number of observed or registered tosses.

Let us now try to extend further this simple example for possible variants involving errors by the observer, and let us imagine what would happen if our observer

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were imperfect and made errors when observing. The observer, for example, might wear thick glasses and have difficulty telling heads from tails, with the result that from time to time, she would incorrectly register a toss as heads or tails, and this would then show up in the resultant probability as a certain error. Because there is only one observer, we automatically, and often even unconsciously, assume that her observations are *exclusive*. Exclusivity means that when our observer registers a toss of heads, he/she automatically does not register a toss of tails, and to the contrary, when registering a toss of tails, he/she does not at the same time register a toss of heads. Thanks to this property, the sum of the resultant probabilities of heads and tails always equals one hundred percent regardless of the size of the observer's error. The error of the observer shows up only by increasing the probability of one side of the coin, while at the same time lowering the probability of the opposite side by the same value.

Now let us assume that we are observing the same phenomenon of coin tossing, but now with two observers who are not consulting each other about their observations. There might be two persons, one of whom watches for and registers only tosses of heads and the other only tails. Let us deal with the geometric interpretation of the errors of two observers. Because the condition must be fulfilled that the length of the hypotenuse of a right triangle must always equal one in Fig.1, we can model the error rates of our observers using the angle between the triangle's legs, so that the square root of the probability determined by the first observer (including his or her errors) will be depicted on the x-axis and the square root of the probability found by the second observer (including that observer's errors) will be depicted on the y-axis.

Unlike classical information science, where the state of a system, or more precisely, information about its state, is described with the use of a probability function, in quantum information science, the information about the state of the system is described using a *complex wave probabilistic function* [5].
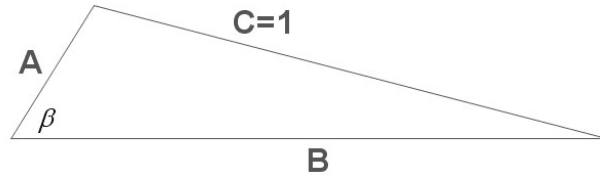
Probability values have analogies to energies and can be modeled as the squares of the values assigned to individual phenomena (concrete values). By the square roots of the probability of event phenomena, one may interpret how dominant a given phenomenon is in a random process, or how often the phenomenon occurs. In this conception, phase indicates the degree of structural links between the individual phenomena, i.e. by analogy the shift with respect to the defined beginning. This beginning may be a phenomenon with a zero phase, to which we relate all of the structural links of the other phenomena.

In accordance with the general principle, we see that we obtain the classical theory of probability by raising the complex wave function to the second power, whereby we automatically lose the phase characteristic of our model.

1.2   Principle of quantization

Let us now assume that we have a certain random phenomenon characterized by its amplitude and phase of wave probabilistic function defined in [5, 6], where these functions typically depend on time or on space. For the sake of clarity, let us assume that our function does not depend on time, but depends rather only on its (x,y) position, i.e. a selected point in an x-y plane. We can also imagine every

Tossing heads: **H**   Tossing experiment: **H,T,T,H,H,T,T,T,H,T**

Tossing tails: **T**   Number of registered tossing: **10**

P observer registered: **2**   O observer registered: **5**

$$p(H) = \frac{2}{10} \qquad\qquad p(T) = \frac{5}{10} \qquad\qquad \beta = 1.076$$

$$C^2 = 1 = A^2 + B^2 \pm 2 \cdot A \cdot B \cdot \cos(\beta) = \left| A + B \cdot e^{j \cdot \beta} \right|^2$$

$$p(H) + p(T) \pm 2 \cdot \sqrt{p(H)} \cdot \sqrt{p(T)} \cdot \cos(\beta) = \left| \sqrt{p(H)} + \sqrt{p(T)} \cdot e^{j \cdot \beta} \right|^2 = 1$$

Fig. 1 *In this non-right-angled triangle, in case of coin tosses, 'C' still must equal 1. 'A' represents the probability of tossing heads as registered by the first observer $p(H)$, and 'B' is the probability of tossing tails as registered by the second observer $p(T)$. The angle β models the errors of the observers.*

point (x,y) in the plane in polar coordinates as a module and phase assigned to that point.

If we change the phase in an x-y plane by multiples of 360 degrees, we are still at our point (x,y), so the resultant amplitude and phase function assigned to the point (x,y) must remain the same for all variant multiples of 360 degrees. If we accept this completely logical requirement for the unambiguousness of the assignment of complex probability to a given point (x,y), we arrive at an important condition for our wave probability function.
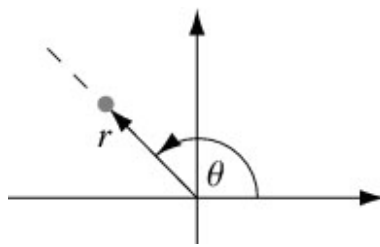
Fig. 2 *The representation of a point (x,y) with polar coordinates using module r and phase θ. Ambiguousness of phase means that for all natural numbers 'n' there is also an identical point for the phase $(\theta + n \cdot 360)$.*

It now suffices to imagine as a trivial example that the phase function of wave probability linearly depends on the phase representation of the position of our (x,y)

point in polar coordinates. This means that the phase parameter of a probability function 'k'' is given by the multiple of the spatial phase assigned to the point (x,y). If the linear multiplying constant 'k'' is not a natural number, in the phase of the probability wave function we get 'k'' multiples of 360 degrees, which lead each time to a different value. The wave probability function takes the form $\psi \propto e^{j \cdot k \cdot (\theta + n \cdot 360)}$, where the symbol $\propto$ means equality up to the normalization factor and j represents an imaginary unit.

These different values of the phase of the wave function mean the ambiguous assigning of a probability wave function to point (x,y). Only in the case that k is a natural number do k multiples of 360 such as 720 or 1080 represent the same value of the phase of the wave probability function.

In this way, we arrive at a simple explanation for the principle of quantization. The phase probability of the wave function must be quantized (k must be a natural number) in order to maintain the unambiguous assigning of a wave probability function to a point (x,y) and to all variants of rotating the coordinates of point (x,y) by multiples of 360 degrees [11]. Or to put it the other way around, we can guarantee the principle of the unambiguous assignment of a complex probability to a given point (x,y) only by the quantization of its phase. The constant k represents a whole-number quantum quasi-spin. In paper [10], I have expanded this principle to any phase functions, and using the Taylor series, I have proven that the quantization of phase is of general validity in wave probability functions.

By the extension of the ideas stated above from an (x,y) plane to an (x,y,z) space, there arises a similar analogy of unambiguousness, but it has more degrees of freedom and leads to expansion of the quantization of quasi-spin. On an (x,y) plane, whole numbered quantum quasi-spin or multiples of the constant k were sufficient, but in an (x,y,z) space, we must introduce halved number values of quantum quasi-spin [6]. The principle of quantization and its justification, however, are identical.

This paper continues in this way of thinking and presents how quasi-spin parameter is important to define probabilistic behavior of quantum systems especially within the process of quantum connection. The paper is structured as follows: Section 2 defines the features of quantum subsystems with different quasi-spins. Section 3 analyses connected two q-bits, and Section 4 covers basic features of connected two quantum sybsystems. Section 5 presents our conclusions.

2. Quantum subsystems with different quasi-spins

2.1 Comparison of classical and quantum probability rules

Let us define quantum binary subsystem marked as η and suppose it can reach two values $|A\rangle_{\eta} = 0$ and $|A\rangle_{\eta} = 1$. With respect to approach presented in Fig. 1 we can suppose that observer No. 1 monitors the state $|A\rangle_{\eta} = 0$ and observer No. 2 the state $|A\rangle_{\eta+1} = 1$.

We assume the phase to be the linear function of quantized phase $m \cdot \Delta$. The phase function must achieve single-valuedness also for the phases $(\Delta + 2 \cdot \pi \cdot k)$ where k is an integer. Mathematically we arrive at following wave probabilistic functions assigned to states $|A\rangle_{\eta} = 0$ and $|A\rangle_{\eta} = 1$:

$$\psi\left(\left|A\right\rangle_\eta = 0\right) = \sqrt{p_0} \tag{1}$$

$$\psi\left(\left|A\right\rangle_{\eta+1} = 1\right) = \sqrt{p_1} \cdot e^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)} \tag{2}$$

where p_0, p_1 are the probabilities of falling zero or one (they are same for both subsystems η and $\eta + 1$).

The probabilities union that $\left|A\right\rangle_\eta = 0$ falling on the subsystem marked η or $\left|A\right\rangle_{\eta+1} = 1$at the second subsystem $\eta + 1$ is given as:

$$P\left(\left(\left|A\right\rangle_\eta = 0\right) \cup \left(\left|A\right\rangle_{\eta+1} = 1\right)\right) = \left|\sqrt{p_0} + \sqrt{p_1} \cdot e^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)}\right|^2 =$$
$$= \quad p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(m \cdot (\Delta + 2 \cdot k \cdot \pi)\right) \tag{3}$$

which is the quantum equivalent of the classical well-known probabilistic rule:

$$P\left(\left(\left|A\right\rangle_\eta = 0\right) \cup \left(\left|A\right\rangle_{\eta+1} = 1\right)\right) =$$
$$= \quad P\left(\left|A\right\rangle_\eta = 0\right) + P\left(\left|A\right\rangle_{\eta+1} = 1\right) - P\left(\left(\left|A\right\rangle_\eta = 0\right) \cap \left(\left|A\right\rangle_{\eta+1} = 1\right)\right) \tag{4}$$

The quantum rule (3) enables both a negative and a positive sign according to a phase parameter. On the other hand, the classical rule (4) enables only a negative sign.

2.2 Information bosons with integer quasi-spin

For *information bosons* with integer quasi-spin $m \in \{0, \pm 1, \pm 2, \pm 3, ...\}$ we can guarantee the positive sign of (3), no matter which value of k is chosen:

$$P\left(\left(\left|A\right\rangle_\eta = 0\right) \cup \left(\left|A\right\rangle_{\eta+1} = 1\right)\right) =$$
$$= \quad p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left[m \cdot (\Delta + 2 \cdot k \cdot \pi)\right] =$$
$$= \quad p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(m \cdot \Delta\right) \tag{5}$$

From intersection rule of wave probabilities [10] we can alternatively write:

$$P\left(\left(\left|A\right\rangle_\eta = 0\right) \cap \left(\left|A\right\rangle_{\eta+1} = 1\right)\right) =$$
$$= \quad \psi^*\left(\left|A\right\rangle_\eta = 0\right) \cdot \psi\left(\left|A\right\rangle_{\eta+1} = 1\right) + \psi\left(\left|A\right\rangle_\eta = 0\right) \cdot \psi^*\left(\left|A\right\rangle_{\eta+1} = 1\right) =$$
$$= \quad 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(m \cdot \Delta + 2 \cdot k \cdot m \cdot \pi\right) = +2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(m \cdot \Delta\right) \tag{6}$$

The final wave probability function for information bosons can be given as:

$$\psi\left(\left|A\right\rangle_\eta = 0\right) = \sqrt{p_0} \tag{7}$$

$$\psi\left(\left|A\right\rangle_{\eta+1} = 1\right) = \sqrt{p_1} \cdot e^{j \cdot m \cdot \Delta} \tag{8}$$

2.3   Information fermions with half-integer quasi-spin

For *information fermions* with half-integer quasi-spin we can find the negative sign in the following way:

$$
P\left(\left(|A\rangle_\eta = 0\right) \cup \left(|A\rangle_{\eta+1} = 1\right)\right) =
$$
$$
= \; p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos[\tfrac{m_1}{2} \cdot (\Delta + 2 \cdot k \cdot \pi)] =
$$
$$
= \; p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos[\tfrac{m_1}{2} \cdot \Delta + m_1 \cdot k \cdot \pi)] =
$$
$$
= \; p_0 + p_1 \pm 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos(\tfrac{m_1}{2} \cdot \Delta) \tag{9}
$$

where $m_1 \in \{\pm 1, \pm 3, \pm 5, ...\}$.

From intersection rule of wave probabilities [10] we can alternatively write:

$$
P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) =
$$
$$
= \; \psi^*\left(|A\rangle_\eta = 0\right) \cdot \psi\left(|A\rangle_{\eta+1} = 1\right) + \psi\left(|A\rangle_\eta = 0\right) \cdot \psi^*\left(|A\rangle_{\eta+1} = 1\right) =
$$
$$
= \; \sqrt{p_0 \cdot p_1} \cdot e^{j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)} + \sqrt{p_0 \cdot p_1} \cdot e^{-j \cdot m \cdot (\Delta + 2 \cdot k \cdot \pi)} =
$$
$$
= \; 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(m \cdot \Delta + 2 \cdot k \cdot m \cdot \pi\right) =
$$
$$
= \; \left\{ \begin{array}{ll} +2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_1}{2} \cdot \Delta\right) & \text{with probability } 1/2 \\ -2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_1}{2} \cdot \Delta\right) & \text{with probability } 1/2 \end{array} \right\} = 0 \tag{10}
$$

It means that the plus and minus probabilities given in (10) cancel each other out and it explains the *exclusion rule* known for indistinguishable fermions.

The final wave probability function for information fermions can be given as:

$$
\psi\left(|A\rangle_\eta = 0\right) = \sqrt{p_0} \tag{11}
$$

$$
\psi\left(|A\rangle_{\eta+1} = 1\right) = \left\{ \begin{array}{l} +\sqrt{p_1} \cdot e^{j \cdot \frac{m_1}{2} \cdot \Delta} \\ -\sqrt{p_1} \cdot e^{j \cdot \frac{m_1}{2} \cdot \Delta} \end{array} \right. \tag{12}
$$

The negative probability was introduced in physics many times e.g. in [15] where it was argued how negative probabilities as well as probabilities above unity possibly could be useful in probability calculations: "If a physical theory for calculation probabilities yields a negative probability for a given situation under certain assumed conditions, we need not conclude the theory is incorrect. Two other possibilities of interpretation exist. One is that conditions (e.g. initial conditions) may not be capable of being realized in the physical world. The other possibility is that the situation for which the probability appears to be negative is not one that can be verified directly. Combination of these two, limitation of verifiability and freedom in initial conditions, may also be a solution to the apparent difficulty"

2.4 Information quarks with third-integer quasi-spin

We can also consider the k-multiple of one-third quasi-spin and we can farther away find the third quantum states assigned to different information quarks as follows:

$$
\begin{aligned}
P&\left(\left(|A\rangle_\eta = 0\right) \cup \left(|A\rangle_{\eta+1} = 1\right)\right) = \\
&= p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos[\tfrac{m_2}{3} \cdot (\Delta + 2 \cdot k \cdot \pi)] = \\
&\; p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos[\tfrac{m_2}{3} \cdot \Delta + \tfrac{m_2}{3} \cdot 2 \cdot k \cdot \pi)] = \\
&= \begin{cases}
p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos(\tfrac{m_2}{3} \cdot \Delta) \\[2mm]
p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \left[-\cos(\tfrac{m_2}{3} \cdot \Delta) + \tfrac{\sqrt{3}}{2} \cdot \sin(\tfrac{m_2}{3} \cdot \Delta)\right] \\[2mm]
p_0 + p_1 + 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \left[-\cos(\tfrac{m_2}{3} \cdot \Delta) - \tfrac{\sqrt{3}}{2} \cdot \sin(\tfrac{m_2}{3} \cdot \Delta)\right]
\end{cases}
\end{aligned} \tag{13}
$$

where $m_2 \in \{\pm 1, \pm 2, \pm 4, \pm 5...\}$.

Alternatively we can rewrite the equation for probabilities intersection as follows:

$$
\begin{aligned}
P&\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) = \\
&= \psi^*\left(|A\rangle_\eta = 0\right) \cdot \psi\left(|A\rangle_{\eta+1} = 1\right) + \psi\left(|A\rangle_\eta = 0\right) \cdot \psi^*\left(|A\rangle_{\eta+1} = 1\right) = \\
&= \sqrt{p_0 \cdot p_1} \cdot e^{j \cdot \frac{m_2}{3} \cdot (\Delta + 2 \cdot k \cdot \pi)} + \sqrt{p_0 \cdot p_1} \cdot e^{-j \cdot \frac{m_2}{3} \cdot (\Delta + 2 \cdot k \cdot \pi)} = \\
&= 2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_2}{3} \cdot \Delta + 2 \cdot k \cdot \tfrac{m_2}{3} \cdot \pi\right) = \\
&= \left\{ \begin{array}{ll}
2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_2}{3} \cdot \Delta\right) & \text{with probability } \tfrac{1}{3} \\
-2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_2}{3} \cdot \Delta\right) + \sqrt{3} \cdot \sqrt{p_0 \cdot p_1} \cdot \sin\left(\tfrac{m_2}{3} \cdot \Delta\right) & \text{with probability } \tfrac{1}{3} \\
-2 \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_2}{3} \cdot \Delta\right) - \sqrt{3} \cdot \sqrt{p_0 \cdot p_1} \cdot \sin\left(\tfrac{m_2}{3} \cdot \Delta\right) & \text{with probability } \tfrac{1}{3}
\end{array} \right\} = \\
&= -\tfrac{2}{3} \cdot \sqrt{p_0 \cdot p_1} \cdot \cos\left(\tfrac{m_2}{3} \cdot \Delta\right)
\end{aligned} \tag{14}
$$

The wave probability function for *information quarks* can be written as:

$$
\psi\left(|A\rangle_\eta = 0\right) = \sqrt{p_0} \tag{15}
$$

$$
\psi\left(|A\rangle_{\eta+1} = 1\right) = \sqrt{p_1} \cdot e^{j \cdot \frac{m_2}{3} \cdot (\Delta + 2 \cdot k \cdot \pi)} = \begin{cases}
\sqrt{p_1} \cdot e^{j \cdot \frac{m_2}{3} \cdot \Delta} \\
\sqrt{p_1} \cdot e^{j \cdot \left(\frac{m_2}{3} \cdot \Delta + \frac{2}{3} \cdot \pi\right)} \\
\sqrt{p_1} \cdot e^{j \cdot \left(\frac{m_2}{3} \cdot \Delta - \frac{2}{3} \cdot \pi\right)}
\end{cases} \tag{16}
$$

It means that three variants of probabilities given in (14) yield into their mutual mixture. It is interesting that the final intersection rule goes to negative probability.

In quarks physics [3] the following two quarks can be found ($e \approx 1.6 \cdot 10^{-19} C$ is the charge of electron):

- u (up-quark) with charge spin $+\tfrac{2}{3} \cdot e$,

- d (down-quark) with charge spin $-\tfrac{1}{3} \cdot e$,

together with their anti-quarks given as:

- $\overline{u}$ (anti-up-quark) with charge spin $-\tfrac{2}{3} \cdot e$,

- $\overline{d}$(anti-down-quark) with charge spin $+\frac{1}{3} \cdot e$.

Other features like quark-flavour or quark-colour come out from three variants of wave function (14). Quarks are recognized as the main components of particle physics and e.g. proton is composed from *uud* quarks (with different colours) and neutron from *ddu* quarks (also with different colours).

The presented introduction of information bosons, fermions or quarks comes from different principle than the well-known ,,standard model of particle physics" [14] that is defined through Lie group U(1)×SU(2)×SU(3). Our methodology is set up only on strict requirement for single-valuedness of wave function.

It is believed that such alternative approach can bring new light into quantum science and can yield into better understanding of connected quantum subsystems, especially for non-integer quasi-spin quantum systems. Such quantum subsystems cannot be found in stable state because the state is continuously switching over all possible (generally infinity) quantum states.

3.   Features of Connected q-bits

Let us compare the quantum and classical probability rules (3) and (4). It is evident that the intersection of probabilities in quantum world can be also negative:

$$P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) < 0 \qquad (17)$$

despite of the fact that probabilities $P\left(|A\rangle_\eta = 0\right) \geq 0, P\left(|A\rangle_{\eta+1} = 1\right) \geq 0$ are positive.

Other probabilities:

$$P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 0\right)\right)$$

$$P\left(\left(|A\rangle_\eta = 1\right) \cap \left(|A\rangle_{\eta+1} = 0\right)\right) \qquad (18)$$

$$P\left(\left(|A\rangle_\eta = 1\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right)$$

should be modified to compensate negative value given at (17). The sum of all possible variants (17) and (18) must be certainly equal to one.

We can distinguish three variants of the intersection of probabilities caused by the introduction of wave probabilistic functions regardless of probabilities' values $P\left(|A\rangle_\eta = 0\right), P\left(|A\rangle_{\eta+1} = 1\right)$:

- the events are *fully independent* – they have no mutual influence:

$$P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) = P\left(|A\rangle_\eta = 0\right) \cdot P\left(|A\rangle_{\eta+1} = 1\right) \qquad (19)$$

- the events are *positively dependent* - they have a mutual positive influence and there exists some "gravitation force" yielding into preference for state $|A\rangle_\eta = 0, |A\rangle_{\eta+1} = 1$ (increasing its probability):

$$P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) > P\left(|A\rangle_\eta = 0\right) \cdot P\left(|A\rangle_{\eta+1} = 1\right) \qquad (20)$$

The maximum probability value is $P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) = 1$, which represents the hundred per cent deterministic occurrence of state $\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)$.

- the events are *negatively dependent* – they have a mutual negative influence and there exists some "gravitation force" yielding against the preference for state $\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)$ (decreasing its probability):

$$P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) \leq 0 \qquad (21)$$

The maximum probability value is $P\left(\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)\right) = 0$, which represents the zero per cent occurrence of state $\left(|A\rangle_\eta = 0\right) \cap \left(|A\rangle_{\eta+1} = 1\right)$.

For information bosons, the principle applies that they mutually attract and cluster into individual spatial areas, and they are the source of energy. On the other hand, the famous Pauli Exclusion Principle applies to information fermions, according to which it is not possible to find two fermions in the same place. Fermions therefore create spatial structures and are responsible for the origin of matter.

4. Features of connected quantum subsystems

Let us have two quantum subsystems A, B described by wave probabilistic functions $\psi_A(.), \psi_B(.)$. First of all we suppose that we are able to distinguish between A and B quantum subsystems. Let us assign features[1] p_1 or p_2 to them. The final quantum system is represented by following wave probabilistic function:

$$\psi_{A,B} = \psi_A(p_1) \cdot \psi_B(p_2). \qquad (22)$$

In case we are not able to assign the right feature to given subsystems A or B we must apply the principle of quantum indistinguishability [10]. It means we have to take into account all variants of possible arrangements:

$$\psi_{A,B} = \psi_A(p_1) \cdot \psi_B(p_2) \pm \psi_A(p_2) \cdot \psi_B(p_1) \qquad (23)$$

where $\pm$ characterizes the symmetry or non-symmetry of both variants (bosons or fermions described above).

Let us suppose that we have the "gravitation energy" between our two subsystems $U_{A,B}(p_1, p_2)$. How many energy will be used for connection between A and B under condition of quantum indistinguishability?

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[1]The feature in our approach is e.g. the special functionality, set of parameters or the part of subsystem. Example of such subsystems is e.g. nucleus of molecule.

From (23) we can compute the probability density:

$$\rho\left(p_1, p_2\right) = \left[\psi_A\left(p_1\right)\right]^2 \cdot \left[\psi_B\left(p_2\right)\right]^2 \pm 2 \cdot \psi_A\left(p_1\right) \cdot \psi_B\left(p_2\right) \cdot \psi_A\left(p_2\right) \cdot \psi_B\left(p_1\right) + \\ + \left[\psi_A\left(p_2\right)\right]^2 \cdot \left[\psi_B\left(p_1\right)\right]^2 \quad (24)$$

The mean value of connection energy is given:

$$\bar{U}_{A,B} \approx C_{A,B} \pm X_{A,B} \quad (25)$$

where $C_{A,B}$ is the classical energy integral and $X_{A,B}$ is the exchange integral that is a consequence of quantum indistinguishability. $C_{A,B}$ and $X_{A,B}$ can be computed by using (24) under symmetry condition [12]:

$$C_{A,B} = \int\limits_{V_1}\int\limits_{V_2} \left[\psi_A\left(p_1\right)\right]^2 \cdot \left[\psi_B\left(p_2\right)\right]^2 \cdot U_{A,B}\left(p_1, p_2\right) \cdot dp_1 \cdot dp_2 \quad (26)$$

$$X_{A,B} = \int\limits_{V_1}\int\limits_{V_2} \psi_A\left(p_1\right) \cdot \psi_B\left(p_2\right) \cdot \psi_A\left(p_2\right) \cdot \psi_B\left(p_1\right) \cdot U_{A,B}\left(p_1, p_2\right) \cdot dp_1 \cdot dp_2 \quad (27)$$

We can mark the distance between two atomic nuclei (subsystems A and B) as $R = |p_1 - p_2|$. Then equation (25) with minus sign represents the valence binding in a hydrogen atom. On this example we can see that the principle of quantum indistinguishability can yield into the understanding of connected quantum subsystems. Fig. 3 describes the energy E of connected two quantum subsystems based on distance R.
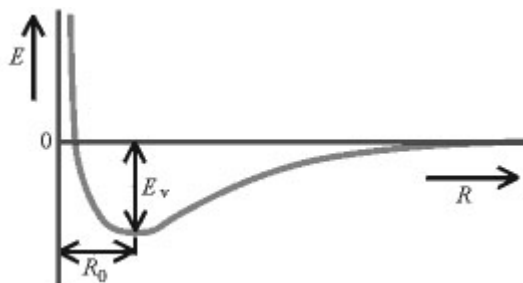
Fig. 3 *The connection energy $E \approx C_{A,B} - X_{A,B}$ of two quantum subsystems.*

This easy example can be extended into more sophisticated links among many quantum subsystems where all possible variants must be taken into consideration to fulfil the principle of quantum indistinguishability.

Let us analyse the solution of Schrodinger wave equation of hydrogen atom. In [12] the detailed method of hydrogen wave function computation is presented. Final solution is composed from many modes. In this paper we will analyse only three modes represented by following wave functions:

$$\psi_1\left(\rho\right) = e^{-\rho} \quad (28)$$

$$\psi_2(\rho) = \left(1 - \frac{\rho}{2}\right) \cdot e^{-\frac{\rho}{2}} \tag{29}$$

$$\psi_3(\rho) = \left(1 - \frac{2 \cdot \rho}{3} + \frac{2}{27} \cdot \rho^2\right) \cdot e^{-\frac{\rho}{3}} \tag{30}$$

where $\rho = \frac{r}{r_B}$ is relative radius, r is radius and $r_B = \frac{\hbar^2}{m \cdot e^2}$ is the constant.
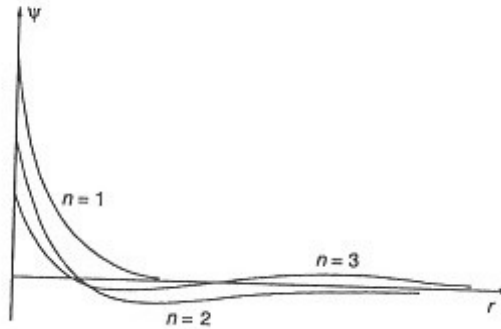
Fig. 4 *Graph representing three variants (for $n = 1, n = 2, n = 3$) of wave functions $\psi_n(r)$ given by equations (28),(29) and (30).*

On Fig. 4 it is clear that wave functions of hydrogen atom could be positive or negative depending on the radius. If we select the radius we can analyse the connection between different modes of wave function. In case both wave functions are positive we can expect "gravitation force" yielding into easy connection of such modes. On the other hand if one wave function is positive and second one is negative it could be expected from the general features of wave functions that there exist "gravitation force" driving our modes away. It is the open question how such above described feature can be practically used.

5. Conclusion

The above described methodology of quantum links can be extended into more complex systems and it is evident that we can explain the way of creation of very complex connections among different quantum subsystems [9].

The presented method can be applied in modelling of macroscopic links amongst complex macroscopic subsystems. As an example of application of our methodology, let us introduce the connections between different companies. We can e.g. imagine that experts from selected companies are members of special committee in which experts create documents based on around the table discussion. People around the table promptly know each other and can easily co-operate even if they work in competitive companies. Round table can be understood as the environment for links creation (more popular word is "networking process"). Networking can be modelled through phase parameters and should represent the quantum links amongst companies. Naturally, the links could be positive or negative according to

relation between experts. It is interesting that such personal links are transformed into real-life relations among companies.

The more personal links, the better relation among companies is set. With respect to such behaviour we can model the natural clusters of companies (system alliances). It is interesting to see how a mathematical instrument of quantum informatics is applicable to modelling of social or business subsystems connections. We believe that quantum system science can improve modelling of soft features of very complex systems like human society. The presented connection principle of quantum subsystems can be for example the powerful instrument for understanding and modelling of system alliances [13].

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